

CHAPTER 10
HYPOTHESIS TESTS INVOLVING
A SAMPLE MEAN OR PROPORTION

SECTION EXERCISES

10.1 d/p/m The null hypothesis is a statement about the value of a population parameter. It is assumed to be true unless we have evidence to the contrary. The alternative hypothesis is an assertion that holds if the null hypothesis is false. The null hypothesis is not always the same as the verbal claim or assertion that led to the test. The null hypothesis must always contain the equal sign. If the directional claim does not contain an equal sign, then the claim is put in the alternative hypothesis and the opposite is put in the null hypothesis.

10.2 d/p/m

- a. Appropriate
- b. Appropriate
- c. Inappropriate. The equal sign must be in the null hypothesis.
- d. Inappropriate. Both hypotheses contain the same signs, the value differs for H_0 and H_1 and the alternative hypothesis contains an equal sign.
- e. Inappropriate. A hypothesis is a statement about a population parameter, not a sample statistic.
- f. Inappropriate. A hypothesis is a statement about a population parameter, not a sample statistic.

10.3 d/p/m

- a. Inappropriate. The value given is different in the two hypotheses.
- b. Appropriate
- c. Appropriate.
- d. Inappropriate. The null and alternative hypotheses do not include all possible values of the population parameter.
- e. Appropriate.
- f. Inappropriate. A hypothesis is a statement about a population parameter, not a sample statistic like p .

10.4 d/p/m The test would be one-tail and the appropriate null and alternative hypotheses would be $H_0: \pi \geq 0.85$ and $H_1: \pi < 0.85$.

10.5 d/p/m If the scientist's null hypothesis that "global warming is taking place" is correct, but people do not take her seriously, they would be making a Type I error by rejecting a true null hypothesis.

10.6 d/p/m H_0 : The person is telling the truth; H_1 : The person is not telling the truth

A Type I error would be committed if we decided the person is not telling the truth when he was telling the truth.

A Type II error would be committed if we decided the person is telling the truth when he was not telling the truth.

10.7 d/p/m The engineer would least like to commit a Type II error since a lot of people could be killed if this occurred. A Type II error would be committed if he decided that the stadium was structurally sound when it was not.

10.8 d/p/m Let π = population proportion of cars that are reported stolen when they were not. Since this is a nondirectional claim, we will use a two-tail test. $H_0: \pi = 0.10$; $H_1: \pi \neq 0.10$

10.9 d/p/m Since this is a directional claim, we will use a one-tail test. $H_0: \pi \leq 0.10$; $H_1: \pi > 0.10$

10.10 d/p/m

- a. $H_0: \mu \leq 300$; $H_1: \mu > 300$ One-tail test b. $H_0: \mu = 1.5$; $H_1: \mu \neq 1.5$ Two-tail test.
c. $H_0: \mu \geq 1200$; $H_1: \mu < 1200$ One-tail test d. $H_0: \mu = 3.5$; $H_1: \mu \neq 3.5$ Two-tail test.

10.11 d/p/m H_0 : Person is not drunk; H_1 : Person is drunk

A Type I error would be committed if the officer decides the person is drunk since he can't walk a straight line or close his eyes and touch his nose, when he really was not drunk. This could occur if a person is tired, frightened, or has a physical disability.

A Type II error would be committed if the officer decides the person is not drunk since he can walk a straight line or close his eyes and touch his nose, but he really is drunk. This could occur because a person drinks quite often and therefore can withstand more.

10.12 d/p/m

- a. In order to NEVER make a Type I error, you would have to always fail to reject H_0 , since a Type I error cannot be made unless you reject H_0 . Therefore, the judge has instructed the jury not to decide the defendant is guilty.
- b. In order to NEVER make a Type II error, you would have to reject H_0 since a Type II error can not be made unless you fail to reject H_0 , Therefore, the judge has instructed the jury not to decide the defendant is innocent.
- c. The jury would try to make the best decision they could from the evidence given. However, they need to remember that they might make a Type I or a Type II error. They would try to minimize the chances of these errors occurring.

10.13 d/p/m She appears to favor Type I error. In this case, Type I error would be deciding that a drug is harmful when it really isn't.

10.14 d/p/m We should use a t-test to carry out the analysis since σ is unknown but we are reasonably sure the population is approximately normally distributed.

10.15 d/p/m Let π = population proportion of women aged 40 - 49 in NYC who save in a 401(k) or individual retirement account. $H_0: \pi = 0.62$ $H_1: \pi \neq 0.62$

This test would be a z-test since $n\pi = 300(0.62) = 186$ and $n(1-\pi) = 300(1 - 0.62) = 114$ are ≥ 5 .

10.16 d/p/m The decision rule specifies the conclusion to be reached for a given outcome of the test (e.g., Reject H_0 if the calculated $z > 1.96$). The decision rule helps us to decide whether to reject H_0 or fail to reject H_0 for a hypothesis test.

10.17 d/p/m The larger the value of α , the greater the likelihood of committing a Type I error.

For this exercise, a Type I error would be deciding the mean tensile strength of the rivets is below 3000 pounds when it is really 3000 pounds or above. With the null and alternative hypotheses,

$H_0: \mu \geq 3000; H_1: \mu < 3000$:

- The marketing director for a major competitor would prefer a numerically high level of significance (e.g., $\alpha = 0.20$) to be used in reaching a conclusion. This would make it easier to conclude that the mean tensile strength is below 3000 pounds when it really is 3000 or above.
- The rivet manufacturer's advertising agency would prefer a numerically low level of significance (e.g., $\alpha = 0.01$) to be used in reaching a conclusion. This would make it more difficult to conclude that the mean tensile strength is below 3000 pounds when it really is 3000 or above. They have already claimed the mean tensile strength to be at least 3000 pounds, so they don't want a test result to suggest otherwise.

10.18 d/p/m Let π = population proportion of defective units. $H_0: \pi \leq 0.05$ $H_1: \pi > 0.05$

This is a one-tail test since this is a directional claim ("no more than 5%"). It is a right-tail test since the alternative hypothesis has a greater-than sign. The rejection region is located in the right tail of the standard normal curve.

10.19 d/p/m If the sample size is large ($n \geq 30$), the central limit theorem assures us that the distribution of sample means will be approximately normally distributed regardless of the shape of the underlying population. The larger the sample size, the better this approximation becomes. When the central limit theorem applies, we may use the standard normal distribution to identify the critical values for the test statistic when σ is known.

10.20 d/p/e If $n < 30$, we must assume that the underlying population is normally distributed in order to use the z-statistic.

10.21 d/p/m A p-value is the exact level of significance associated with the calculated value of the test statistic. It is the most extreme critical value that the test statistic would be capable of exceeding.

If p-value $< \alpha$, reject H_0 and if p-value $\geq \alpha$, do not reject H_0 .

10.22 d/p/m Since p-value = 0.03 is less than $\alpha = 0.05$, the null hypothesis would be rejected. The sample result is more extreme than you would have been willing to attribute to chance.

10.23 d/p/m Since p-value = 0.04 is not less than $\alpha = 0.01$, the null hypothesis would be not be rejected. The sample result is not more extreme than you would have been willing to attribute to chance.

10.24 d/p/m If we are unable to reject H_0 , then the p-value is not less than the level of significance being used ($\alpha = 0.01$), or p-value ≥ 0.01 .

10.25 c/a/m Using the standard normal table,

- p-value = $P(z \geq 1.54) = 1.0000 - 0.9382 = 0.0618$
- p-value = $P(z \leq -1.03) = 0.1515$
- p-value = $2P(z \leq -1.83) = 2(0.0336) = 0.0672$

10.26 c/a/m Using the standard normal table,

- p-value = $P(z \leq -1.62) = 0.0526$
- p-value = $P(z \geq 1.43) = 1.0000 - 0.9236 = 0.0764$
- p-value = $2P(z \geq 1.27) = 2(1.0000 - 0.8980) = 2(0.1020) = 0.2040$

10.27 c/a/m Null and alternative hypotheses:

$H_0: \mu = 450$ $H_1: \mu \neq 450$ Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 458$, $n = 35$ (known: $\sigma = 20.5$)

$$\text{Calculated value of test statistic: } z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{458 - 450}{20.5/\sqrt{35}} = 2.31$$

Critical values: $z = -1.96$ and $z = 1.96$ (in the normal distribution, the area between $z = -1.96$ and $z = 1.96$ is 0.95, and the sum of the two tail areas is 0.05).

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96 , otherwise do not reject.

Conclusion: Since calculated test statistic falls in rejection region ($z = 2.31 > 1.96$), reject H_0 .

Decision: At the 0.05 level, the results suggest that the population mean is not 450.

Using the standard normal distribution table, we can find the approximate p-value as twice the area to the right of $z = 2.31$. This is $2(1.0000 - 0.9896) = 2(0.0104) = 0.0208$.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. The results are shown below. For this two-tail test, the p-value (0.0210) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. For a true null hypothesis, there is only a 0.0210 probability that a sample mean this far away from 450 would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	458.0	z Stat	2.31
4	Population standard deviation	20.5	P(Z<=z) one-tail	0.0105
5	Sample size	35	z Critical one-tail	1.645
6	Hypothesized mean	450	P(Z<=z) two-tail	0.0210
7	Alpha	0.05	z Critical two-tail	1.960

10.28 c/a/m Null and alternative hypotheses:

$H_0: \mu \leq 220$ $H_1: \mu > 220$ Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 230.8$, $n = 12$ (known: $\sigma = 17$ and the population is normally distributed.)

$$\text{Calculated value of test statistic: } z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{230.8 - 220}{17/\sqrt{12}} = 2.20$$

Critical value: $z = 1.645$ (in the normal distribution, the area to the right of $z = 1.645$ is 0.05).

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Since calculated test statistic falls in rejection region ($z = 2.20 > 1.645$), reject H_0 .

Decision: At the 0.05 level, the results suggest that the population mean is greater than 220.

Using the standard normal distribution table, we can find the approximate p-value as the area to the right of $z = 2.20$. This is $1.0000 - 0.9861 = 0.0139$.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. The results are shown below. For this right-tail test, the p-value (0.0139) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. For a true null hypothesis, there is only a 0.0139 probability that a sample mean this much larger than 220 would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	230.8	z Stat	2.20
4	Population standard deviation	17	P(Z<=z) one-tail	0.0139
5	Sample size	12	z Critical one-tail	1.645
6	Hypothesized mean	220	P(Z<=z) two-tail	0.0278
7	Alpha	0.05	z Critical two-tail	1.960

10.29 p/a/m Null and alternative hypotheses:

$H_0: \mu = 2$ (machine in adjustment) $H_1: \mu \neq 2$ (machine out of adjustment)

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 2.025$, $n = 35$ (known: $\sigma = 0.07$)

Calculated value of test statistic: $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{2.025 - 2}{0.07 / \sqrt{35}} = 2.11$

Critical values: $z = -2.58$ and $z = 2.58$ (in the normal distribution, the area between $z = -2.58$ and $z = 2.58$ is 0.99, and the sum of the two tail areas is 0.01).

Decision rule: Reject H_0 if the calculated $z < -2.58$ or > 2.58 , otherwise do not reject.

Conclusion: Since calculated test statistic falls in nonrejection region ($-2.58 < z = 2.11 < 2.58$) do not reject H_0 .

Decision: At the 0.01 level, results suggest the machine is properly adjusted. It appears the mean length of nails produced by the machine could be 2 inches. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

Using the standard normal distribution table, we can find the approximate p-value as twice the area to the right of $z = 2.11$. This is $2(1.0000 - 0.9826) = 2(0.0174) = 0.0348$.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. For this two-tail test, the p-value (0.0346) is not less than the 0.01 level of significance being used to reach a conclusion, so the null hypothesis is not rejected. For a true null hypothesis, there is a 0.0346 probability that a sample mean this far away from 2.000 inches would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	2.025	z Stat	2.11
4	Population standard deviation	0.07	P(Z<=z) one-tail	0.0173
5	Sample size	35	z Critical one-tail	2.326
6	Hypothesized mean	2.000	P(Z<=z) two-tail	0.0346
7	Alpha	0.01	z Critical two-tail	2.576

10.30 p/a/m Null and alternative hypotheses:

$H_0: \mu \geq 5.00$ (no decline in spending) $H_1: \mu < 5.00$ (a decline)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 4.20$, $n = 18$ (known: $\sigma = 1.80$ and the population is normally distributed.)

Calculated value of test statistic: $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{4.20 - 5.00}{1.80 / \sqrt{18}} = -1.89$

Critical value: $z = -1.645$ (in the normal distribution the area to the left of $z = -1.645$ is 0.05).

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the results suggest a decline in spending on popcorn and snacks at the cinema complex. It appears the average amount spent is now less than \$5.00.

Using the standard normal distribution table, we can find the approximate p-value as the area to the left of $z = -1.89$. This is 0.0294.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. For this left-tail test, the p-value (0.0297) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. For a true null hypothesis, there is only a 0.0297 probability that a sample mean this much less than \$5.00 would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	4.20	z Stat	-1.89
4	Population standard deviation	1.80	P(Z<=z) one-tail	0.0297
5	Sample size	18	z Critical one-tail	1.645
6	Hypothesized mean	5.00	P(Z<=z) two-tail	0.0593
7	Alpha	0.05	z Critical two-tail	1.960

10.31 p/a/m Null and alternative hypotheses:

$H_0: \mu = 2.5$ (machine doesn't need maintenance) $H_1: \mu \neq 2.5$ (needs maintenance)

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 2.509$, $n = 34$ (known: $\sigma = 0.027$)

Calculated value of test statistic: $z = \frac{\bar{x} - \mu_0}{\frac{\sigma_x}{\sqrt{n}}} = \frac{2.509 - 2.50}{0.027 / \sqrt{34}} = 1.94$

Critical values: $z = -2.58$ and $z = 2.58$ (in the normal distribution, the area between $z = -2.58$ and $z = 2.58$ is 0.99, and the sum of the two tail areas is 0.01).

Decision rule: Reject H_0 if the calculated $z < -2.58$ or > 2.58 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.01 level, the results suggest that the machine is not in need of maintenance and calibration. The mean diameter of the tubing appears to still be 2.5 inches. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

Using the standard normal distribution table, we can find the approximate p-value as twice the area to the right of $z = 1.94$. This is $2(1.0000 - 0.9738) = 2(0.0262) = 0.0524$.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. For this two-tail test, the p-value (0.0519) is not less than the 0.01 level of significance being used to reach a conclusion, so the null hypothesis is not rejected. For a true null hypothesis, there is a 0.0519 probability that a sample mean this far away from 2.500 would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	2.509	z Stat	1.94
4	Population standard deviation	0.027	P(Z<=z) one-tail	0.0260
5	Sample size	34	z Critical one-tail	2.326
6	Hypothesized mean	2.500	P(Z<=z) two-tail	0.0519
7	Alpha	0.01	z Critical two-tail	2.576

10.32 p/a/m Null and alternative hypotheses:

$H_0: \mu \geq 3$ (new booklet does not reduce assembly time) $H_1: \mu < 3$ (reduces assembly time)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 2.90$, $n = 15$ (known: $\sigma = 0.20$ and the population is normally distributed)

$$\text{Calculated value of test statistic: } z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2.90 - 3.00}{0.20/\sqrt{15}} = -1.94$$

Critical value: $z = -1.645$ (in the normal distribution the area to the left of $z = -1.645$ is 0.05).

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the new booklet appears to be effective in reducing the time for an inexperienced kit builder to assemble the device. The mean time for assembly with the new booklet is less than 3 hours.

Using the standard normal distribution table, we can find the approximate p-value as the area to the left of $z = -1.94$. This is 0.0262.

Given the summary data, we can also carry out this z-test using the Tests Statistics workbook that accompanies Data Analysis Plus. For this left-tail test, the p-value (0.0264) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. For a true null hypothesis, there is only a 0.0264 probability that a sample mean this much less than 3.00 hours would occur by chance.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	2.90	z Stat	-1.94
4	Population standard deviation	0.20	P(Z<=z) one-tail	0.0264
5	Sample size	15	z Critical one-tail	1.645
6	Hypothesized mean	3.00	P(Z<=z) two-tail	0.0528
7	Alpha	0.05	z Critical two-tail	1.960

10.33 p/c/m The null and alternative hypotheses are $H_0: \mu = \$10,526$ and $H_1: \mu \neq \$10,526$.

The Data Analysis Plus and Minitab results are shown below.

	A	B	C	D
1	Z-Test: Mean			
2				
3				<i>price</i>
4	Mean			10842.95
5	Standard Deviation			1667.09
6	Observations			40
7	Hypothesized Mean			10526
8	SIGMA			2000
9	z Stat			1.00
10	P(Z<=z) one-tail			0.1581
11	z Critical one-tail			1.645
12	P(Z<=z) two-tail			0.3162
13	z Critical two-tail			1.96

One-Sample Z: price

Test of mu = 10526 vs not = 10526
The assumed standard deviation = 2000

Variable	N	Mean	StDev	SE Mean	95% CI	Z	P
price	40	10843	1667	316	(10223, 11463)	1.00	0.316

For this two-tail test, the p-value (0.316) is not less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is not rejected. At this level of significance, we conclude that the

mean price for home office remodeling in this region could be the same as the mean price for the nation as a whole.

10.34 p/c/m The null and alternative hypotheses are $H_0: \mu = 70$ pounds and $H_1: \mu \neq 70$ pounds. The Data Analysis Plus and Minitab results are shown below.

	A	B	C	D
1	Z-Test: Mean			
2				
3				/lbs.
4	Mean			69.61
5	Standard Deviation			1.081
6	Observations			35
7	Hypothesized Mean			70
8	SIGMA			1.0
9	z Stat			-2.307
10	P(Z<=z) one-tail			0.011
11	z Critical one-tail			1.645
12	P(Z<=z) two-tail			0.021
13	z Critical two-tail			1.960

One-Sample Z: Lbs.

Test of mu = 70 vs mu not = 70
The assumed sigma = 1

Variable	N	Mean	StDev	SE Mean
Lbs.	35	69.610	1.081	0.169

Variable	95.0% CI	Z	P
Lbs.	(69.279, 69.941)	-2.31	0.021

For this two-tail test, the p-value (0.021) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. At this level of significance, we conclude that the mean fill weight for the machine could be something other than 70.0 pounds. For a true null hypothesis, there would be only a 0.021 probability of obtaining a sample mean this far away from 70.0 pounds just by chance.

10.35 d/p/m

- Do not reject H_0 since 170 is in the 90% confidence interval given.
- Reject H_0 since 110 is not in the 90% confidence interval given.
- Do not reject H_0 since 130 is in the 90% confidence interval given.
- Reject H_0 since 200 is not in the 90% confidence interval given.

10.36 c/a/m From exercise 10.27, $\bar{x} = 458.0$, $\sigma = 20.5$, $n = 35$, the critical z values for a two-tail test at the $\alpha = 0.05$ level are $z = -1.96$ and $z = 1.96$, and the hypothesis test is $H_0: \mu = 450$ versus $H_1: \mu \neq 450$. The 95% confidence interval for μ is:

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 458.0 \pm 1.96 \frac{20.5}{\sqrt{35}} = 458.0 \pm 6.79, \text{ or from } 451.21 \text{ to } 464.79$$

Since 450 is not in the 95% confidence interval for μ found above, the population mean is probably not equal to 450. In exercise 10.27, the null hypothesis was rejected and we concluded that the population mean is not equal to 450. Therefore, the conclusion using the confidence interval is the same as the conclusion from the hypothesis test. The confidence interval can also be obtained using the Estimators workbook that accompanies Data Analysis Plus, as shown below.

	A	B	C	D	E
1	z-Estimate of a Mean				
2					
3	Sample mean	458.0	Confidence Interval Estimate		
4	Population standard deviation	20.5	458.00	±	6.79
5	Sample size	35	Lower confidence limit		451.21
6	Confidence level	0.95	Upper confidence limit		464.79

10.37 c/a/m From exercise 10.29, $\bar{x} = 2.025$ inches, $\sigma = 0.070$ inches, $n = 35$, the critical z values for a two-tail test at the $\alpha = 0.01$ level are $z = -2.58$ and $z = 2.58$, and the hypothesis test is $H_0: \mu = 2.000$ versus $H_1: \mu \neq 2.000$. The 99% confidence interval for μ is:

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 2.025 \pm 2.58 \frac{0.070}{\sqrt{35}} = 2.025 \pm 0.031, \text{ or from } 1.994 \text{ to } 2.056$$

Since 2.000 is within the 99% confidence interval for μ found above, the population mean could be equal to 2.000. In exercise 10.29, the null hypothesis was not rejected and we concluded that the population mean could be 2.000. Therefore, the conclusion using the confidence interval is the same as the conclusion from the hypothesis test. The confidence interval can also be obtained using the Estimators workbook that accompanies Data Analysis Plus, as shown below. Because it does not rely on the printed standard normal table (with its gaps between listed values), this interval is more accurate, and has lower and upper limits of 1.995 inches and 2.055 inches, respectively.

	A	B	C	D	E
1	z-Estimate of a Mean				
2					
3	Sample mean	2.025	Confidence Interval Estimate		
4	Population standard deviation	0.07	2.025	±	0.030
5	Sample size	35	Lower confidence limit		1.995
6	Confidence level	0.99	Upper confidence limit		2.055

10.38 c/a/m From exercise 10.31, $\bar{x} = 2.509$ inches, $\sigma = 0.027$ inches, $n = 34$, the critical z values for a two-tail test at the $\alpha = 0.01$ level are $z = -2.58$ and $z = 2.58$, and the hypothesis test is $H_0: \mu = 2.500$ versus $H_1: \mu \neq 2.500$. The 99% confidence interval for μ is:

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 2.509 \pm 2.58 \frac{0.027}{\sqrt{34}} = 2.509 \pm 0.012, \text{ or from } 2.497 \text{ to } 2.521$$

Since 2.500 is within the 99% confidence interval for μ found above, the population mean could be equal to 2.500. In exercise 10.31, the null hypothesis was not rejected and we concluded that the population mean could be 2.500. Therefore, the conclusion using the confidence interval is the same as the conclusion from the hypothesis test. The confidence interval can also be obtained using the Estimators workbook that accompanies Data Analysis Plus, as shown below.

	A	B	C	D	E
1	z-Estimate of a Mean				
2					
3	Sample mean	2.509	Confidence Interval Estimate		
4	Population standard deviation	0.027	2.509	±	0.012
5	Sample size	34	Lower confidence limit		2.497
6	Confidence level	0.99	Upper confidence limit		2.521

10.39 d/p/e The t statistic should be used in carrying out a hypothesis test for the mean when σ is unknown. When $n < 30$, we must assume the population is approximately normally distributed.

10.40 c/a/m Null and alternative hypotheses: $H_0: \mu = 24.0$ $H_1: \mu \neq 24.0$

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 25.9$, $s = 4.2$, $n = 40$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{25.9 - 24.0}{4.2/\sqrt{40}} = 2.861$$

Critical values: $t = -2.708$ and $t = 2.708$ For this test, $\alpha = 0.01$ and d.f. = $(n - 1) = (40 - 1) = 39$.

Referring to the $0.01/2 = 0.005$ column and the 39th row of the t table, the critical values are $t = -2.708$ and $t = 2.708$.

Decision rule: Reject H_0 if the calculated $t < -2.708$ or > 2.708 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.01 level, the results suggest that the population mean is not equal to 24.0.

10.41 c/a/m Null and alternative hypotheses: $H_0: \mu \geq 90.0$ $H_1: \mu < 90.0$

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 82.0$, $s = 20.5$, $n = 15$ (Note: population is approximately normally distributed.)

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{82.0 - 90.0}{20.5/\sqrt{15}} = -1.511$$

Critical value: $t = -1.761$. For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (15 - 1) = 14$. Referring to the 0.05 column and the 14th row of the t table, the critical value is $t = -1.761$.

Decision rule: Reject H_0 if the calculated $t < -1.761$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, the results suggest that the population mean could be at least 90.0.

The sample mean could have been this low merely by chance.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this left-tail test, the p-value (0.076) is not less than 0.05, so we do not reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	82.0	t Stat	-1.511
4	Sample standard deviation	20.5	P(T<=t) one-tail	0.076
5	Sample size	15	t Critical one-tail	1.761
6	Hypothesized mean	90.0	P(T<=t) two-tail	0.153
7	Alpha	0.05	t Critical two-tail	2.145

10.42 p/a/m Null and alternative hypotheses:

$H_0: \mu \leq 9.0$ (employees' cars no older than national average) $H_1: \mu > 9.0$ (cars are older)

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 10.4$, $s = 3.1$, $n = 34$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{10.4 - 9.0}{3.1/\sqrt{34}} = 2.633$$

Critical value: $t = 2.445$ For this test, $\alpha = 0.01$ and d.f. = $(n - 1) = (34 - 1) = 33$. Referring to the 0.01 column and the 33rd row of the t table, the critical value is $t = 2.445$.

Decision rule: Reject H_0 if the calculated $t > 2.445$, otherwise do not reject.

Conclusion: Calculated test statistic falls into the rejection region, reject H_0 .

Decision: At the 0.01 level, we conclude that the average age of cars driven to work by the plant's employees could be more than the national average of 9.0 years.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this right-tail test, the p-value (0.006) is less than 0.05, so we are able to reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	10.4	t Stat	2.633
4	Sample standard deviation	3.1	P(T<=t) one-tail	0.006
5	Sample size	34	t Critical one-tail	2.445
6	Hypothesized mean	9	P(T<=t) two-tail	0.013
7	Alpha	0.01	t Critical two-tail	2.733

10.43 p/a/m Null and alternative hypotheses:

$H_0: \mu = 464$ (the average flight is 464 miles, the value reported by the industry association) and

$H_1: \mu \neq 464$ (the average flight is not 464 miles) Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 479.6$, $s = 42.8$, $n = 30$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{479.6 - 464}{42.8/\sqrt{30}} = 1.996$$

Critical values: $t = -2.045$ and $t = 2.045$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (30 - 1) = 29$.

Referring to the $0.05/2 = 0.025$ column and the 29th row of the t table, the critical values are $t = -2.045$ and $t = 2.045$.

Decision rule: Reject H_0 if the calculated $t < -2.045$ or > 2.045 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, the results do not cause us to doubt that the average length of a flight by regional airlines in the U.S. is the reported value, 464 miles. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.055) is not less than 0.05, so we do not reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	479.6	t Stat	1.996
4	Sample standard deviation	42.8	P(T<=t) one-tail	0.028
5	Sample size	30	t Critical one-tail	1.699
6	Hypothesized mean	464	P(T<=t) two-tail	0.055
7	Alpha	0.05	t Critical two-tail	2.045

10.44 p/a/m Null and alternative hypotheses:

$H_0: \mu = 1.65$ (the mean daily coffee consumption in this city is the same as for all U.S. residents)

$H_1: \mu \neq 1.65$ (the mean daily coffee consumption in this city differs from the overall U.S.)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 1.84$, $s = 0.85$, $n = 38$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{1.84 - 1.65}{0.85/\sqrt{38}} = 1.378$$

Critical values: $t = -2.026$ and $t = 2.026$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (38 - 1) = 37$.

Referring to the $0.05/2 = 0.025$ column and the 37th row of the t table, the critical values are $t = -2.026$ and $t = 2.026$.

Decision rule: Reject H_0 if the calculated $t < -2.026$ or > 2.026 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, the mean daily coffee consumption for the residents of this North Carolina city does not differ significantly from their counterparts across the nation. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.177) is not less than 0.05, so we do not reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	1.84	t Stat	1.378
4	Sample standard deviation	0.85	P(T<=t) one-tail	0.088
5	Sample size	38	t Critical one-tail	1.687
6	Hypothesized mean	1.65	P(T<=t) two-tail	0.177
7	Alpha	0.05	t Critical two-tail	2.026

10.45 p/a/m Null and alternative hypotheses:

$H_0: \mu = 150$ (Taxco's assertion is accurate) $H_1: \mu \neq 150$ (assertion is not accurate)

Level of significance: $\alpha = 0.10$

Test results: $\bar{x} = 125$, $s = 43$, $n = 12$ (assumed: population is approximately normally distributed)

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{125 - 150}{43/\sqrt{12}} = -2.014$$

Critical values: $t = -1.796$ and $t = 1.796$ For this test, $\alpha = 0.10$ and d.f. = $(n - 1) = (12 - 1) = 11$.

Referring to the $0.10/2 = 0.05$ column and the 11th row of the t table, the critical values are $t = -1.796$ and $t = 1.796$.

Decision rule: Reject H_0 if the calculated $t < -1.796$ or > 1.796 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.10 level, the results suggest that Taxco's assertion that the mean refund for those customers who received refunds last year was \$150 is not accurate.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.069) is less than 0.10, so we are able to reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	125.0	t Stat	-2.014
4	Sample standard deviation	43	P(T<=t) one-tail	0.035
5	Sample size	12	t Critical one-tail	1.363
6	Hypothesized mean	150.0	P(T<=t) two-tail	0.069
7	Alpha	0.10	t Critical two-tail	1.796

10.46 p/a/m Null and alternative hypotheses:

$H_0: \mu = 8.7$ (mean length of membership is 8.7 years) $H_1: \mu \neq 8.7$ (mean length is not 8.7 yrs.)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 7.2$, $s = 2.5$, $n = 15$ (assumed: population is approximately normally distributed)

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_x} = \frac{7.2 - 8.7}{2.5/\sqrt{15}} = -2.324$$

Critical values: $t = -2.145$ and $t = 2.145$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (15 - 1) = 14$.

Referring to the $0.05/2 = 0.025$ column and the 14th row of the t table, the critical values are $t = -2.145$ and $t = 2.145$.

Decision rule: Reject H_0 if the calculated $t < -2.145$ or > 2.145 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the results suggest that the actual mean length of membership may be some value other than 8.7 years.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.036) is less than 0.05, so we are able to reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	7.2	t Stat	-2.324
4	Sample standard deviation	2.5	P(T<=t) one-tail	0.018
5	Sample size	15	t Critical one-tail	1.761
6	Hypothesized mean	8.7	P(T<=t) two-tail	0.036
7	Alpha	0.05	t Critical two-tail	2.145

10.47 p/a/d Null and alternative hypotheses:

$H_0: \mu \leq 80$ (the mean of cash sales is no more than \$80)

$H_1: \mu > 80$ (the mean of cash sales is greater than \$80)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 91$, $s = 21$, $n = 20$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_x} = \frac{91 - 80}{21/\sqrt{20}} = 2.343$$

Critical value: $t = 1.729$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (20 - 1) = 19$. Referring to the 0.05 column and the 19th row of the t table, the critical value is $t = 1.729$.

Decision rule: Reject H_0 if the calculated $t > 1.729$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, it appears that the agent's suspicion is confirmed. The mean of the scrap metal dealer's cash sales appears to exceed \$80.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this right-tail test, the p-value (0.015) is less than 0.05, so we are able to reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	91.0	t Stat	2.343
4	Sample standard deviation	21.0	P(T<=t) one-tail	0.015
5	Sample size	20	t Critical one-tail	1.729
6	Hypothesized mean	80.0	P(T<=t) two-tail	0.030
7	Alpha	0.05	t Critical two-tail	2.093

10.48 p/a/m Null and alternative hypotheses:

$H_0: \mu \leq 1478$ (the average earnings at this university is not higher than the national mean)

$H_1: \mu > 1478$ (the average earnings at this university is higher than the national mean)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 1503, s = 210, n = 45$

Calculated value of test statistic: $t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{1503 - 1478}{210/\sqrt{45}} = 0.799$

Critical value: $t = 1.680$ For this test, $\alpha = 0.05$ and d.f. $= (n - 1) = (45 - 1) = 44$. Referring to the 0.05 column and the 44th row of the t table, the critical value is $t = 1.680$.

Decision rule: Reject H_0 if the calculated $t > 1.680$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, the results suggest that the average earnings of this university's work-study students are not higher than the national average of \$1478. The sample mean could have been this high merely by chance.

Given the summary data, we can also carry out this test using the Test Statistics workbook that accompanies Data Analysis Plus. In this right-tail test, the p-value (0.214) is not less than 0.05, so we are not able to reject the null hypothesis.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	1503	t Stat	0.799
4	Sample standard deviation	210	P(T<=t) one-tail	0.214
5	Sample size	45	t Critical one-tail	1.680
6	Hypothesized mean	1478	P(T<=t) two-tail	0.429
7	Alpha	0.05	t Critical two-tail	2.015

10.49 p/a/m Null and alternative hypotheses: $H_0: \mu = \$640,000$ and $H_1: \mu \neq \$640,000$

The exercise can be solved by hand, but we will use the computer and the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.147) is not less than 0.05, so we do not reject the null hypothesis. At the 0.05 level of significance, the mean for the older households in this region may be the same as the national mean.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	615000	t Stat	-1.473
4	Sample standard deviation	120000	P(T<=t) one-tail	0.074
5	Sample size	50	t Critical one-tail	1.677
6	Hypothesized mean	640000	P(T<=t) two-tail	0.147
7	Alpha	0.05	t Critical two-tail	2.010

10.50 p/a/m Using the Estimators workbook that accompanies Data Analysis Plus, we obtain the 95% confidence interval shown below. We are 95% confident the mean for older households in this region is within the interval from \$580,896 to \$649,104. Because the hypothesized mean for this region (\$640,000) is within the interval, we conclude that the mean for this region could be \$640,000. This is the same conclusion reached in the hypothesis test of exercise 10.49.

	A	B	C	D	E
1	t-Estimate of a Mean				
2					
3	Sample mean	615000	Confidence Interval Estimate		
4	Sample standard deviation	120000	615000	±	34104
5	Sample size	50	Lower confidence limit		580896
6	Confidence level	0.95	Upper confidence limit		649104

10.51 p/a/m Null and alternative hypotheses: $H_0: \mu \geq 4000$ hours and $H_1: \mu < 4000$ hours
The exercise can be solved by hand, but we will use the computer and the Test Statistics workbook that accompanies Data Analysis Plus. In this left-tail test, the p-value (0.019) is less than 0.025, so we reject H_0 and conclude that the conditions may be having an adverse effect on bulb life.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	3882	t Stat	-2.285
4	Sample standard deviation	200	P(T<=t) one-tail	0.019
5	Sample size	15	t Critical one-tail	2.145
6	Hypothesized mean	4000	P(T<=t) two-tail	0.038
7	Alpha	0.025	t Critical two-tail	2.510

10.52 p/a/m Null and alternative hypotheses: $H_0: \mu = 93$ minutes and $H_1: \mu \neq 93$ minutes
The exercise can be solved by hand, but we will use the computer and the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.033) is less than 0.05, so we reject H_0 and conclude that the population mean is some value other than 93 minutes.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	89.5	t Stat	-2.190
4	Sample standard deviation	11.3	P(T<=t) one-tail	0.017
5	Sample size	50	t Critical one-tail	1.677
6	Hypothesized mean	93	P(T<=t) two-tail	0.033
7	Alpha	0.05	t Critical two-tail	2.010

10.53 p/a/m Using the Estimators workbook, we obtain the 95% confidence interval shown below. We are 95% confident the population mean is within the interval from 86.29 to 92.71 seconds.
Because the hypothesized population mean (93 minutes) is not within this interval, we conclude that the actual population mean must be some value other than 93 minutes. This is the same conclusion reached in the hypothesis test of exercise 10.52.

	A	B	C	D	E
1	t-Estimate of a Mean				
2					
3	Sample mean	89.5	Confidence Interval Estimate		
4	Sample standard deviation	11.3	89.50	plus/minus	3.21
5	Sample size	50	Lower confidence limit		86.29
6	Confidence level	0.95	Upper confidence limit		92.71

10.54 p/a/m Null and alternative hypotheses: $H_0: \mu = 38$ minutes and $H_1: \mu \neq 38$ minutes
The exercise can be solved by hand, but we will use the computer and the Test Statistics workbook that accompanies Data Analysis Plus. In this two-tail test, the p-value (0.085) is less than 0.10, so we reject H_0 and conclude that the actual population mean time for completion might be some value other than 38 minutes.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	36.8	t Stat	-1.775
4	Sample standard deviation	4	P(T<=t) one-tail	0.042
5	Sample size	35	t Critical one-tail	1.307
6	Hypothesized mean	38	P(T<=t) two-tail	0.085
7	Alpha	0.10	t Critical two-tail	1.691

10.55 p/a/m Using the Estimators workbook, we obtain the 90% confidence interval shown below. We are 90% confident the population mean completion time is within the interval from 35.657 minutes to 37.943 minutes. Because the hypothesized population mean (38 minutes) is not within this interval, we conclude that the actual population mean must be some value other than 38 minutes. This is the same conclusion reached in the hypothesis test of exercise 10.54.

	A	B	C	D	E
1	t-Estimate of a Mean				
2					
3	Sample mean	36.8	Confidence Interval Estimate		
4	Sample standard deviation	4.0	36.800	plus/minus	1.143
5	Sample size	35	Lower confidence limit		35.657
6	Confidence level	0.90	Upper confidence limit		37.943

10.56 p/c/m The null and alternative hypotheses are $H_0: \mu \leq \$57$ and $H_1: \mu > \$57$. The Data Analysis Plus and Minitab results are shown below.

	A	B	C	D
1	t-Test: Mean			
2				
3				<i>Spent</i>
4	Mean			61.05
5	Standard Deviation			14.54
6	Hypothesized Mean			57
7	df			44
8	t Stat			1.869
9	P(T<=t) one-tail			0.034
10	t Critical one-tail			2.015
11	P(T<=t) two-tail			0.068
12	t Critical two-tail			2.321

One-Sample T: Spent

Test of mu = 57 vs > 57

2.5%

Variable	N	Mean	StDev	SE Mean	Lower Bound	T	P
Spent	45	61.05	14.54	2.17	65.42	1.87	0.034

For this right-tail test, the p-value (0.034) is not less than the 0.025 level of significance being used to reach a conclusion, so the null hypothesis is not rejected. At this level of significance, we conclude that the mean mall shopping expenditure for teens in this area may not be any higher than for U.S. teens as a whole.

10.57 p/c/m The null and alternative hypotheses are $H_0: \mu = \$817$ and $H_1: \mu \neq \$817$. The Data Analysis Plus and Minitab results are shown below.

	A	B	C	D
1	t-Test: Mean			
2				
3				<i>Expense</i>
4	Mean			850.58
5	Standard Deviation			136.05
6	Hypothesized Mean			817
7	df			79
8	t Stat			2.207
9	P(T<=t) one-tail			0.015
10	t Critical one-tail			1.664
11	P(T<=t) two-tail			0.030
12	t Critical two-tail			1.991

One-Sample T: Expense

Test of mu = 817 vs not = 817

Variable	N	Mean	StDev	SE Mean	95% CI	T	P
Expense	80	850.6	136.1	15.2	(820.3, 880.9)	2.21	0.030

For this two-tail test, the p-value (0.030) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. If the North Carolina mean were really \$817, there would be only a 0.030 probability of obtaining a sample mean this far away from \$817. We conclude that the mean for North Carolina motorists is some value other than \$817.

10.58 p/c/m As shown in the Minitab printout in the solution to exercise 10.57, the 95% confidence interval for the North Carolina mean is from \$820.3 to \$880.9. The hypothesized mean (\$817) is not within the interval, so we conclude that the mean for North Carolina must be some value other than \$817. This is the same conclusion that was reached in exercise 10.57.

10.59 d/p/e The normal distribution is a good approximation for the binomial distribution when $n\pi$ and $n(1-\pi)$ are both ≥ 5 .

10.60 c/a/m Null and alternative hypotheses: $H_0: \pi = 0.40$ $H_1: \pi \neq 0.40$

Level of significance: $\alpha = 0.01$

Test results: $p = 0.34$, $n = 200$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.34 - 0.40}{\sqrt{0.4(1 - 0.4) / 200}} = -1.73$

Critical values: $z = -2.58$ and $z = 2.58$

Decision rule: Reject H_0 if the calculated $z < -2.58$ or > 2.58 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.01 level, the results suggest that the population proportion could be 0.40.

The difference between the hypothesized population proportion and the sample proportion is judged to have been merely the result of chance variation.

Given the summary data, we can also use the Test Statistics workbook that accompanies Data Analysis Plus. For this two-tail test, the p-value (0.083) is not less than the 0.01 level of significance being used to reach a conclusion, so do not reject the null hypothesis.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.34	z Stat	-1.73
4	Sample size	200	P(Z<=z) one-tail	0.042
5	Hypothesized proportion	0.40	z Critical one-tail	2.326
6	Alpha	0.01	P(Z<=z) two-tail	0.083
7			z Critical two-tail	2.576

10.61 c/a/m Null and alternative hypotheses: $H_0: \pi \geq 0.50$ $H_1: \pi < 0.50$

Level of significance: $\alpha = 0.05$

Test results: $p = 0.47$, $n = 1000$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.47 - 0.50}{\sqrt{0.5(1 - 0.5)/1000}} = -1.90$

Critical value: $z = -1.645$

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the results suggest that the population proportion is less than 0.50.

Given the summary data, we can also use the Test Statistics workbook that accompanies

Data Analysis Plus. For this left-tail test, the p-value (0.029) is less than the 0.05 level of significance being used to reach a conclusion, so reject the null hypothesis.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.47	z Stat	-1.90
4	Sample size	1000	P(Z<=z) one-tail	0.029
5	Hypothesized proportion	0.50	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.058
7			z Critical two-tail	1.960

10.62 c/a/m Null and alternative hypotheses: $H_0: \pi \leq 0.60$ $H_1: \pi > 0.60$

Level of significance: $\alpha = 0.025$

Test results: $p = 0.63$, $n = 700$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.63 - 0.60}{\sqrt{0.6(1 - 0.6)/700}} = 1.62$

Critical value: $z = 1.96$

Decision rule: Reject H_0 if the calculated $z > 1.96$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.025 level, the results suggest that the population proportion is no more than 0.60.

The sample proportion could have been this large merely by chance.

10.63 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.02$ (the proportion of defectives is no more than 0.02)

$H_1: \pi > 0.02$ (the proportion of defectives is greater than 0.02)

Level of significance: We will use $\alpha = 0.05$ in carrying out this right-tail test.

Test results: $p = 0.04$, $n = 300$

$$\text{Calculated value of test statistic: } z = \frac{p - \pi_0}{\sigma_p} = \frac{0.04 - 0.02}{\sqrt{0.02(1 - 0.02)/300}} = 2.47$$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the results suggest that the supplier's claim is not correct. The true percentage of defectives in the shipment appears to be greater than 2%.

Given the summary data, we can also use the Test Statistics workbook that accompanies

Data Analysis Plus. For this right-tail test, the p-value (0.007) is less than the 0.05 level of significance being used to reach a conclusion, so reject the null hypothesis.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.04	z Stat	2.47
4	Sample size	300	P(Z<=z) one-tail	0.007
5	Hypothesized proportion	0.02	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.013
7			z Critical two-tail	1.960

10.64 p/a/m Null and alternative hypotheses:

$H_0: \pi = 0.15$ (the proportion of juniors who apply for admission is 0.15)

$H_1: \pi \neq 0.15$ (the proportion of juniors who apply for admission is not 0.15)

Level of significance: $\alpha = 0.05$

Test results: $p = 30/300 = 0.10$, $n = 300$

$$\text{Calculated value of test statistic: } z = \frac{p - \pi_0}{\sigma_p} = \frac{0.10 - 0.15}{\sqrt{0.15(1 - 0.15)/300}} = -2.43$$

Critical values: $z = -1.96$ and $z = 1.96$

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the results suggest that we should reject the director's claim. The true proportion of high school juniors to whom she sends university literature who eventually apply for admission is not 0.15.

10.65 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.05$ (the proportion who violated the agreement is no more than 0.05)

$H_1: \pi > 0.05$ (the proportion who violated the agreement is more than 0.05)

Level of significance: $\alpha = 0.025$

Test results: $p = 0.08$, $n = 400$

$$\text{Calculated value of test statistic: } z = \frac{p - \pi_0}{\sigma_p} = \frac{0.08 - 0.05}{\sqrt{0.05(1 - 0.05)/400}} = 2.75$$

Critical value: $z = 1.96$

Decision rule: Reject H_0 if the calculated $z > 1.96$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.025 level, the data do not support the human resource's director's claim that no more than 5% of employees hired in the past year have violated their pre-employment agreement not to use any of five illegal drugs.

Given the summary data, we can also use the Test Statistics workbook that accompanies Data Analysis Plus. For this right-tail test, the p-value (0.003) is less than the 0.025 level of significance being used to reach a conclusion, so reject the null hypothesis.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.08	z Stat	2.75
4	Sample size	400	P(Z<=z) one-tail	0.003
5	Hypothesized proportion	0.05	z Critical one-tail	1.960
6	Alpha	0.025	P(Z<=z) two-tail	0.006
7			z Critical two-tail	2.241

10.66 p/a/m Null and alternative hypotheses:

$H_0: \pi = 0.65$ (percentage who prefer electric heating has not changed)

$H_1: \pi \neq 0.65$ (percentage who prefer electric heating has changed)

Level of significance: $\alpha = 0.05$

Test results: $p = 0.60$, $n = 200$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.60 - 0.65}{\sqrt{0.65(1 - 0.65) / 200}} = -1.48$

Critical values: $z = -1.96$ and $z = 1.96$

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, we cannot conclude that the percentage of residential energy consumers who prefer to heat with electricity instead of gas has changed from 65%. The difference between the hypothesized population proportion and the sample proportion is judged to have been merely the result of chance variation.

The p-value for this two-tail test is twice the area to the left of $z = -1.48$, or $2(0.0694) = 0.1388$.

Because the p-value is not less than 0.05, we do not reject the null hypothesis. Using the Test Statistics workbook, the corresponding results are shown below.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.60	z Stat	-1.48
4	Sample size	200	P(Z<=z) one-tail	0.069
5	Hypothesized proportion	0.65	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.138
7			z Critical two-tail	1.960

10.67 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.44$ (the proportion passing on the first try has not increased)

$H_1: \pi > 0.44$ (the proportion passing on the first try has increased)

Level of significance: $\alpha = 0.05$

Test results: $p = 130/250 = 0.52$, $n = 250$

$$\text{Calculated value of test statistic: } z = \frac{p - \pi_0}{\sigma_p} = \frac{0.52 - 0.44}{\sqrt{0.44(1 - 0.44)/250}} = 2.55$$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, we can conclude that the proportion passing on the first try has increased from 0.44.

The p-value for this right-tail test is the area to the right of $z = 2.55$, or $1.0000 - 0.9946 = 0.0054$. Because the p-value is less than 0.05, we reject the null hypothesis. Using the Test Statistics workbook, the corresponding results are shown below.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.52	z Stat	2.55
4	Sample size	250	P(Z<=z) one-tail	0.005
5	Hypothesized proportion	0.44	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.011
7			z Critical two-tail	1.960

10.68 p/a/m Null and alternative hypotheses:

$H_0: \pi \geq 0.66$ (the proportion who have purchased life insurance is at least 0.66)

$H_1: \pi < 0.66$ (the proportion who have purchased life insurance is less than 0.66)

Level of significance: $\alpha = 0.05$

Test results: $p = 0.56$, $n = 50$

$$\text{Calculated value of test statistic: } z = \frac{p - \pi_0}{\sigma_p} = \frac{0.56 - 0.66}{\sqrt{0.66(1 - 0.66)/50}} = -1.49$$

Critical value: $z = -1.645$

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, the sample finding is not significantly lower than the 66% reported by the research firm for the U.S. overall.

The p-value for this left-tail test is the area to the left of $z = -1.49$, or 0.0681.

Because the p-value is not less than 0.05, we do not reject the null hypothesis. Using the Test Statistics workbook, the corresponding results are shown below.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.56	z Stat	-1.49
4	Sample size	50	P(Z<=z) one-tail	0.068
5	Hypothesized proportion	0.66	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.136
7			z Critical two-tail	1.960

10.69 p/a/m The null and alternative hypotheses are $H_0: \pi = 0.55$ and $H_1: \pi \neq 0.55$.

This solution can be obtained with a pocket calculator and formulas, but we will use the computer. As shown in the Test Statistics printout for this two-tail test, the p-value (0.014) is less than the 0.05 level of significance being used to reach a conclusion, so reject the null hypothesis. If the population proportion for this builder were really 0.55, there would be only a 0.014 probability of obtaining a sample proportion this far away from 0.55.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.50	z Stat	-2.46
4	Sample size	600	P(Z<=z) one-tail	0.007
5	Hypothesized proportion	0.55	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.014
7			z Critical two-tail	1.960

10.70 p/a/m This solution can be obtained with a pocket calculator and formulas, but we will use the computer. As shown in the Estimators printout below, the 95% confidence interval for the population proportion for this builder is from 0.460 to 0.540. The hypothesized proportion (0.55) is not within the interval, so we conclude that the proportion for this builder must be some value other than 0.55. This is the same conclusion that was reached in exercise 10.69.

	A	B	C	D	E
1	z-Estimate of a Proportion				
2					
3	Sample proportion	0.50	Confidence Interval Estimate		
4	Sample size	600	0.50	±	0.040
5	Confidence level	0.95	Lower confidence limit		
6			Upper confidence limit		
					0.460
					0.540

10.71 p/a/m The null and alternative hypotheses are $H_0: \pi = 0.07$ and $H_1: \pi \neq 0.07$.

This solution can be obtained with a pocket calculator and formulas, but we will use the computer.

As shown in the Test Statistics printout for this two-tail test, the p-value (0.220) is not less than the 0.10 level of significance used to reach a conclusion, so we do not reject the null hypothesis.

The percentage of young women who are low-paid in this county might be the same as the percentage of young woman who are low-paid in the nation as a whole.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.084	z Stat	1.23
4	Sample size	500	P(Z<=z) one-tail	0.110
5	Hypothesized proportion	0.07	z Critical one-tail	1.282
6	Alpha	0.10	P(Z<=z) two-tail	0.220
7			z Critical two-tail	1.645

10.72 p/a/m This solution can be obtained with a pocket calculator and formulas, but we will use the computer. As shown in the Estimators printout below, the 90% confidence interval for the population

proportion for this county is from 0.064 to 0.104. The hypothesized proportion (0.07) is within the interval, so we conclude that the population proportion of young women who are low-paid in this county could be 0.07. This is the same conclusion that was reached in exercise 10.71.

	A	B	C	D	E
1	z-Estimate of a Proportion				
2					
3	Sample proportion	0.084	Confidence Interval Estimate		
4	Sample size	500	0.084	±	0.020
5	Confidence level	0.90	Lower confidence limit		0.064
6			Upper confidence limit		0.104

10.73 p/a/m The null and alternative hypotheses are $H_0: \pi \leq 0.50$ and $H_1: \pi > 0.50$.

This solution can be obtained with a pocket calculator and formulas, but we will use the computer.

As shown in the Test Statistics printout for this right-tail test, the p-value (0.079) is not less than the 0.025 level of significance being used to reach a conclusion, so we do not reject the null hypothesis.

The sample proportion is not significantly greater than the 0.50 value we would expect simply by chance.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.55	z Stat	1.41
4	Sample size	200	P(Z<=z) one-tail	0.079
5	Hypothesized proportion	0.50	z Critical one-tail	1.960
6	Alpha	0.025	P(Z<=z) two-tail	0.157
7			z Critical two-tail	2.241

10.74 p/a/m The null and alternative hypotheses are $H_0: \pi = 0.80$ and $H_1: \pi \neq 0.80$.

This solution can be obtained with a pocket calculator and formulas, but we will use the computer.

As shown in the Test Statistics printout for this two-tail test, the p-value (0.134) is not less than the 0.10 level of significance used to reach a conclusion, so we do not reject the null hypothesis.

The auditor's performance does not differ significantly from the hypothesized 0.80 value.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.77	z Stat	-1.50
4	Sample size	400	P(Z<=z) one-tail	0.067
5	Hypothesized proportion	0.80	z Critical one-tail	1.282
6	Alpha	0.10	P(Z<=z) two-tail	0.134
7			z Critical two-tail	1.645

10.75 p/a/m This solution can be obtained with a pocket calculator and formulas, but we will use the computer. As shown in the Estimators printout below, the 90% confidence interval for the population proportion for this auditor is from 0.735 to 0.805. The hypothesized proportion (0.80) is within the

interval, so we conclude that the population proportion for this auditor could be 0.80. This is the same conclusion that was reached in exercise 10.74.

	A	B	C	D	E
1	z-Estimate of a Proportion				
2					
3	Sample proportion	0.77	Confidence Interval Estimate		
4	Sample size	400	0.770	±	0.035
5	Confidence level	0.90	Lower confidence limit		0.735
6			Upper confidence limit		0.805

10.76 p/c/m The null and alternative hypotheses are $H_0: \pi = 0.41$ and $H_1: \pi \neq 0.41$.

As shown in the Data Analysis Plus printout for this two-tail test, the p-value (0.151) is not less than the 0.10 level of significance, so we do not reject the null hypothesis. The graduation rate for male basketball players from this region could be the same as the rate for their counterparts nationwide.

	A	B	C	D
1	z-Test: Proportion			
2				
3				Status
4	Sample Proportion			0.46
5	Observations			200
6	Hypothesized Proportion			0.41
7	z Stat			1.438
8	P(Z<=z) one-tail			0.075
9	z Critical one-tail			1.282
10	P(Z<=z) two-tail			0.151
11	z Critical two-tail			1.645

10.77 p/c/m As shown in the Data Analysis Plus printout, the 90% confidence interval for the population graduation rate for male basketball players from this region is from 0.402 to 0.518. Because the hypothesized proportion (0.41) is within this interval, we conclude that the population proportion for this region could be 0.41. This is the same as the conclusion that was reached in exercise 10.76.

	A	B
1	z-Estimate: Proportion	
2		Status
3	Sample Proportion	0.460
4	Observations	200
5	LCL	0.402
6	UCL	0.518

10.78 p/c/m The null and alternative hypotheses are $H_0: \pi \leq 0.35$ and $H_1: \pi > 0.35$.

As shown in the Data Analysis Plus printout for this right-tail test, the p-value (0.035) is less than the 0.05 level of significance used to reach a conclusion, so we reject the null hypothesis. The high rate of visitors in the sample who went to the Can Do page is too large to have occurred by chance.

	A	B	C	D
1	z-Test: Proportion			
2				
3				<i>Visited</i>
4	Sample Proportion			0.40
5	Observations			300
6	Hypothesized Proportion			0.35
7	z Stat			1.816
8	P(Z<=z) one-tail			0.035
9	z Critical one-tail			1.645
10	P(Z<=z) two-tail			0.069
11	z Critical two-tail			1.960

10.79 d/p/m The power of a test is the probability that the test will respond correctly by rejecting a false null hypothesis. By calculating the power of the test ($1 - \beta$) for several assumed values for the population mean and plotting the power versus the population mean, we arrive at the power curve. By looking at the power curve, you can get an idea of how powerful the hypothesis test is for different possible values of the population mean.

10.80 d/p/m The operating characteristic (OC) curve plots the probability that the hypothesis test will NOT reject the null hypothesis for assumed values for the population mean. The OC curve is the complement of the power curve. It is found by plotting β versus the population mean.

10.81 d/p/m Alpha has already been specified as 0.05 so, when the sample size is increased, α will stay the same and β will be decreased for the test.

10.82 p/a/d From exercise 10.31, $\sigma = 0.027$, $n = 34$, $\sigma_{\bar{x}} = 0.00463$, the hypothesis test is

$H_0: \mu = 2.5$ versus $H_1: \mu \neq 2.5$, and the decision rule is "Reject H_0 if the calculated test statistic $z < -2.58$ or > 2.58 ."

First, get the decision rule in terms of \bar{x} .

Sample mean, \bar{x} , corresponding to critical $z = -2.58$ is $2.5 - 2.58(0.00463) = 2.488$

Sample mean, \bar{x} , corresponding to critical $z = 2.58$ is $2.5 + 2.58(0.00463) = 2.512$

The decision rule in terms of \bar{x} is: "Reject H_0 if $\bar{x} < 2.488$ or > 2.512 "

Next, convert these sample means into z values using the true mean of $\mu = 2.52$.

$$\text{when } \bar{x} = 2.488, \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.52}{0.00463} = -6.91$$

$$\text{when } \bar{x} = 2.512, \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.52}{0.00463} = -1.73$$

$$\beta = P(-6.91 \leq z \leq -1.73) = 0.0418 - 0.0000 = 0.0418$$

$$\text{Power of the test} = 1 - \beta = 1 - 0.0418 = 0.9582$$

Using the Beta-means workbook that accompanies Data Analysis Plus, we get a comparable result. Refer to the "Two-tail Test" worksheet and enter the requisite information into cells B3:B7.

The power of the test is shown in D6.

	A	B	C	D
1	Type II Error			
2				
3	H0: MU	2.500	Critical values	2.49
4	SIGMA	0.027		2.51
5	Sample size	34	Prob(Type II error)	0.04
6	ALPHA	0.01	Power of the test	0.96
7	H1: MU	2.520		

10.83 p/a/d From exercise 10.32, $\sigma = 0.20$, $n = 15$, $\sigma_{\bar{x}} = 0.05164$, the hypothesis test is $H_0: \mu \geq 3$ versus $H_1: \mu < 3$, and the decision rule is "Reject H_0 if the calculated $z < -1.645$."

First, get the decision rule in terms of \bar{x} .

Sample mean, \bar{x} , corresponding to critical $z = -1.645$ is $3 - 1.645(0.05164) = 2.915$

The decision rule in terms of \bar{x} is: "Reject H_0 if $\bar{x} < 2.915$ "

Next, convert the sample mean into a z value using the true mean of $\mu = 2.80$.

$$\text{when } \bar{x} = 2.915, \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 2.80}{0.05164} = 2.23$$

$$\beta = P(z \geq 2.23) = 1.0000 - 0.9871 = 0.0129$$

$$\text{Power of the test} = 1 - \beta = 1 - 0.0129 = 0.9871$$

Using the Beta-means workbook that accompanies Data Analysis Plus, we get a comparable result. Refer to the "Left-tail Test" worksheet and enter the requisite information into cells B3:B7.

The power of the test is shown in D5

	A	B	C	D
1	Type II Error			
2				
3	H0: MU	3.00	Critical value	2.92
4	SIGMA	0.20	Prob(Type II error)	0.0129
5	Sample size	15	Power of the test	0.9871
6	ALPHA	0.05		
7	H1: MU	2.80		

10.84 p/a/d From exercise 10.31, $\sigma = 0.027$, $n = 34$, and $\sigma_{\bar{x}} = 0.00463$. From exercise 10.82, the decision rule in terms of \bar{x} is "Reject H_0 if $\bar{x} < 2.488$ or > 2.512 ."

Now, find $1 - \beta$ for each assumed true population mean given.

Given $\mu = 2.485$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.485}{0.00463} = 0.65$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.485}{0.00463} = 5.83$$

$$\text{and } 1 - \beta = 1 - P(0.65 \leq z \leq 5.83) = 1 - (1.0000 - 0.7422) = 1 - 0.2578 = 0.7422$$

Given $\mu = 2.490$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.490}{0.00463} = -0.43$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.490}{0.00463} = 4.75$$

$$\text{and } 1 - \beta = 1 - P(-0.43 \leq z \leq 4.75) = 1 - (1.0000 - 0.3336) = 1 - 0.6664 = 0.3336$$

Given $\mu = 2.495$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.495}{0.00463} = -1.51$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.495}{0.00463} = 3.67$$

$$\text{and } 1 - \beta = 1 - P(-1.51 \leq z \leq 3.67) = 1 - (1.0000 - 0.0655) = 1 - 0.9345 = 0.0655$$

Given $\mu = 2.500$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.500}{0.00463} = -2.59$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.500}{0.00463} = 2.59$$

$$\text{and } 1 - \beta = 1 - P(-2.59 \leq z \leq 2.59) = 1 - (0.9952 - 0.0048) = 1 - 0.9904 = 0.0096$$

Given $\mu = 2.505$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.505}{0.00463} = -3.67$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.505}{0.00463} = 1.51$$

$$\text{and } 1 - \beta = 1 - P(-3.67 \leq z \leq 1.51) = 1 - (0.9345 - 0.0000) = 1 - 0.9345 = 0.0655$$

Given $\mu = 2.510$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.510}{0.00463} = -4.75$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.510}{0.00463} = 0.43$$

$$\text{and } 1 - \beta = 1 - P(-4.75 \leq z \leq 0.43) = 1 - (0.6664 - 0.0000) = 1 - 0.6664 = 0.3336$$

Given $\mu = 2.515$:

$$\text{when } \bar{x} = 2.488, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.488 - 2.515}{0.00463} = -5.83$$

$$\text{when } \bar{x} = 2.512, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.512 - 2.515}{0.00463} = -0.65$$

$$\text{and } 1 - \beta = 1 - P(-5.83 \leq z \leq -0.65) = 1 - (0.2578 - 0.0000) = 1 - 0.2578 = 0.7422$$

Using Minitab, the following results are obtained. The "Difference" column refers to the difference between the assumed actual μ and the value of μ in the null hypothesis.

For example, the Difference = -0.15 row shows the power of the test when the assumed actual mean is 2.485 inches compared to the hypothesized mean of 2.500 inches.

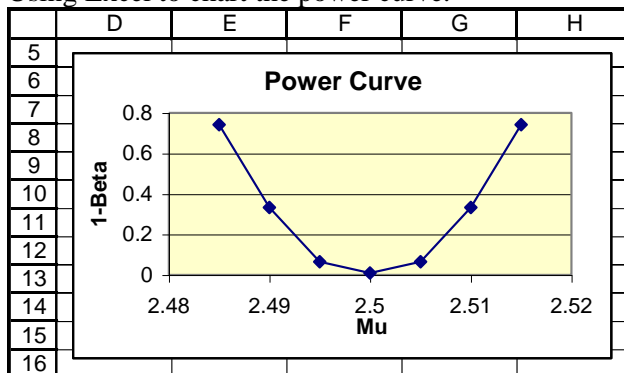
Power and Sample Size

1-Sample Z Test

Testing mean = null (versus not = null)
 Calculating power for mean = null + difference
 Alpha = 0.01 Sigma = 0.027

Difference	Sample Size	Power
-0.015	34	0.7465
-0.010	34	0.3386
-0.005	34	0.0675
0.000	34	0.0100
0.005	34	0.0675
0.010	34	0.3386
0.015	34	0.7465

Using Excel to chart the power curve.



10.85 p/a/d From exercise 10.32, $\sigma = 0.20$, $n = 15$, and $\sigma_{\bar{x}} = 0.05164$. From exercise 10.83, the decision rule in terms of \bar{x} is "Reject H_0 if $\bar{x} < 2.915$."

Now, find $1 - \beta$ for each assumed true population mean given.

Given $\mu = 2.80$:

$$\text{when } \bar{x} = 2.915, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 2.80}{0.05164} = 2.23$$

$$\text{and } 1 - \beta = 1 - P(z \geq 2.23) = 1 - (1.0000 - 0.9871) = 1 - 0.0129 = 0.9871$$

Given $\mu = 2.85$:

$$\text{when } \bar{x} = 2.915, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 2.85}{0.05164} = 1.26$$

$$\text{and } 1 - \beta = 1 - P(z \geq 1.26) = 1 - (1.0000 - 0.8962) = 1 - 0.1038 = 0.8962$$

Given $\mu = 2.90$:

$$\text{when } \bar{x} = 2.915, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 2.90}{0.05164} = 0.29$$

$$\text{and } 1 - \beta = 1 - P(z \geq 0.29) = 1 - (1.0000 - 0.6141) = 1 - 0.3859 = 0.6141$$

Given $\mu = 2.95$:

$$\text{when } \bar{x} = 2.915, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 2.95}{0.05164} = -0.68$$

$$\text{and } 1 - \beta = 1 - P(z \geq -0.68) = 1 - (1.0000 - 0.2483) = 1 - 0.7517 = 0.2483$$

Given $\mu = 3.00$:

$$\text{when } \bar{x} = 2.915, z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{2.915 - 3.00}{0.05164} = -1.65$$

$$\text{and } 1 - \beta = 1 - P(z \geq -1.65) = 1 - (1.0000 - 0.0495) = 1 - 0.9505 = 0.0495$$

Using Minitab, the following results are obtained. The “Difference” column refers to the difference between the assumed actual μ and the value of μ in the null hypothesis.

For example, the Difference = -0.20 row shows the power of the test when the assumed actual mean is 2.80 hours compared to the hypothesized mean of 3.00 hours.

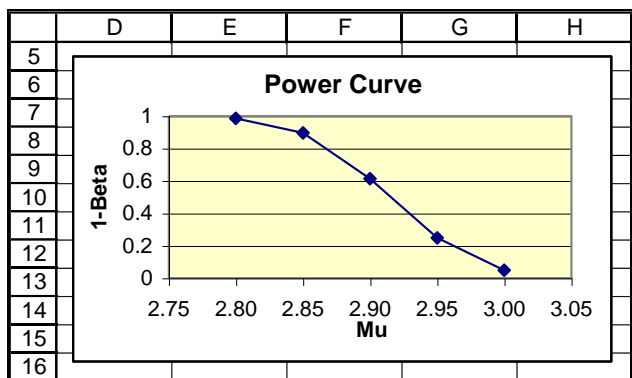
Power and Sample Size

1-Sample Z Test

Testing mean = null (versus < null)
Calculating power for mean = null + difference
Alpha = 0.05 Sigma = 0.2

Difference	Sample Size	Power
-0.20	15	0.9871
-0.15	15	0.8961
-0.10	15	0.6147
-0.05	15	0.2493
0.00	15	0.0500

Using Excel to chart the power curve.



10.86 p/a/d From exercise 10.63, $p = 0.04$, $n = 300$, the hypothesis test is $H_0: \pi \leq 0.02$ versus $H_1: \pi > 0.02$, and the decision rule is "Reject H_0 if the calculated $z > 1.645$." The standard error of p can be calculated as:

$$\sigma_p = \sqrt{\frac{0.02(1-0.02)}{300}} = 0.00808$$

First, get the decision rule in terms of p .

Sample proportion corresponding to critical $z = 1.645$ is $0.02 + 1.645(0.00808) = 0.033$

The decision rule in terms of p will be: "Reject H_0 if $p > 0.033$."

Now, find $1 - \beta$ for each assumed true population proportion given.

Given $\pi = 0.02$:

$$\text{when } p = 0.033, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.02}{0.00808} = 1.61$$

$$\text{and } 1 - \beta = 1 - P(z \leq 1.61) = 1 - 0.9463 = 0.0537$$

Note: By definition, this should be 0.0500, but it differs due to rounding errors. See the Minitab note that follows the calculations.

Given $\pi = 0.03$:

$$\text{when } p = 0.033, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.03}{0.00808} = 0.37$$

$$\text{and } 1 - \beta = 1 - P(z \leq 0.37) = 1 - 0.6443 = 0.3557$$

Given $\pi = 0.04$:

$$\text{when } p = 0.033, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.04}{0.00808} = -0.87$$

$$\text{and } 1 - \beta = 1 - P(z \leq -0.87) = 1 - 0.1922 = 0.8078$$

Given $\pi = 0.05$:

$$\text{when } p = 0.033, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.05}{0.00808} = -2.10$$

$$\text{and } 1 - \beta = 1 - P(z \leq -2.10) = 1 - 0.0179 = 0.9821$$

Given $\pi = 0.06$:

$$\text{when } p = 0.033, z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.06}{0.00808} = -3.34$$

$$\text{and } 1 - \beta = 1 - P(z \leq -3.34) = 1 - 0.0000 = 1.0000$$

Given $\pi = 0.07$:

$$\text{when } p = 0.033, z = \frac{p - \pi}{\sigma_p} = \frac{0.033 - 0.07}{0.00808} = -4.58$$

$$\text{and } 1 - \beta = P(z \leq -4.58) = 1 - 0.0000 = 1.0000$$

Using Minitab, note that the “Alternative Proportion” column refers to the assumed actual π and the entries are in scientific notation – e.g., “2.00E-02” represents 2.00×10^{-2} , or 0.02. The Minitab results are much more accurate than the ones calculated above, largely due to our rounding in the quantities either leading to the calculation or resulting from it, including p , σ_p , and z .

Power and Sample Size

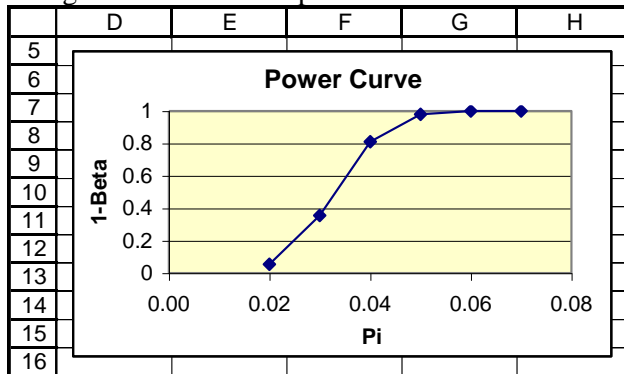
Test for One Proportion

Testing proportion = 0.02 (versus > 0.02)

Alpha = 0.05

Alternative Proportion	Sample Size	Power
2.00E-02	300	0.0500
3.00E-02	300	0.3690
4.00E-02	300	0.7233
5.00E-02	300	0.9078
6.00E-02	300	0.9743
7.00E-02	300	0.9936

Using Excel to chart the power curve.



10.87 p/a/d $H_0: \pi \leq 0.02$ versus $H_1: \pi > 0.02$, with $\alpha = 0.01$, $n = 400$, and the standard error of p calculated as:

$$\sigma_p = \sqrt{\frac{0.02(1-0.02)}{400}} = 0.007$$

- $z = 2.33$ for a right-tail area of 0.01 beneath the normal curve.
- Getting the decision rule in terms of p : $p = 0.02 + 2.33(0.007) = 0.036$
and the decision rule is: "Reject H_0 if $p > 0.036$."
- $\beta = P(\text{fail to reject a false } H_0)$

Given $\pi = 0.02$:

$$\text{when } p = 0.036, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.036 - 0.02}{0.007} = 2.29$$

$$\text{and } \beta = P(z \leq 2.29) = 0.9890$$

Given $\pi = 0.03$:

$$\text{when } p = 0.036, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.036 - 0.03}{0.007} = 0.86$$

$$\text{and } \beta = P(z \leq 0.86) = 0.8051$$

Given $\pi = 0.04$:

$$\text{when } p = 0.036, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.036 - 0.04}{0.007} = -0.57$$

$$\text{and } \beta = P(z \leq -0.57) = 0.2843$$

Given $\pi = 0.05$:

$$\text{when } p = 0.036, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.036 - 0.05}{0.007} = -2.00$$

$$\text{and } \beta = P(z \leq -2.00) = 0.0228$$

Given $\pi = 0.06$:

$$\text{when } p = 0.036, \quad z = \frac{p - \pi}{\sigma_p} = \frac{0.036 - 0.06}{0.007} = -3.43$$

$$\text{and } \beta = P(z \leq -3.43) = 0.0000$$

- Using the calculations carried out in part c,
 - When $\pi = 0.02$, $1 - \beta = 1 - 0.9890 = 0.0110$
 - When $\pi = 0.03$, $1 - \beta = 1 - 0.8051 = 0.1949$
 - When $\pi = 0.04$, $1 - \beta = 1 - 0.2843 = 0.7157$
 - When $\pi = 0.05$, $1 - \beta = 1 - 0.0228 = 0.9772$
 - When $\pi = 0.06$, $1 - \beta = 1 - 0.0000 = 1.0000$

Using Minitab, note that the "Alternative Proportion" column refers to the assumed actual π and the entries are in scientific notation -- e.g., "2.00E-02" represents 2.00×10^{-2} , or 0.02. The Minitab results are much more accurate than the ones calculated above, largely due to our rounding in the quantities either leading to the calculation or resulting from it, including p , σ_p , and z .

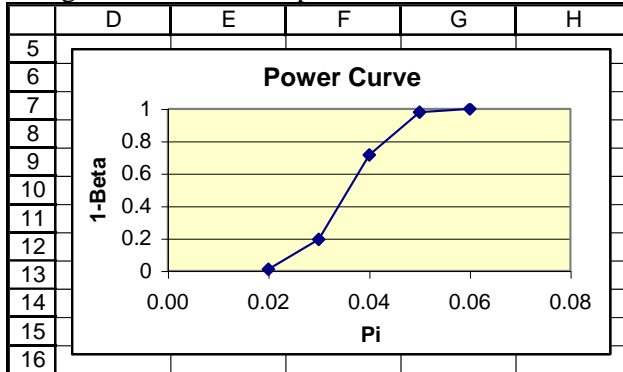
Power and Sample Size

Test for One Proportion

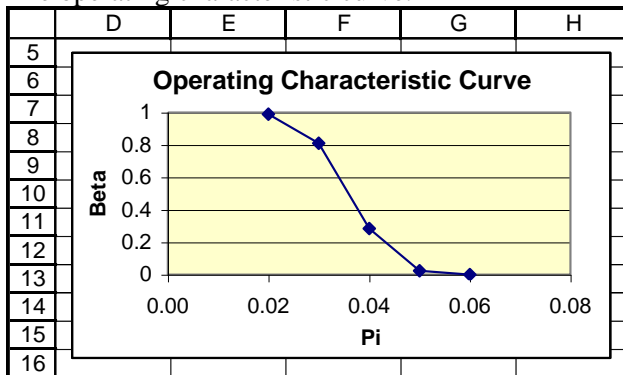
Testing proportion = 0.02 (versus > 0.02)
Alpha = 0.01

Alternative Proportion	Sample Size	Power
2.00E-02	400	0.0100
3.00E-02	400	0.2306
4.00E-02	400	0.6477
5.00E-02	400	0.8959
6.00E-02	400	0.9771

Using Excel to chart the power curve.



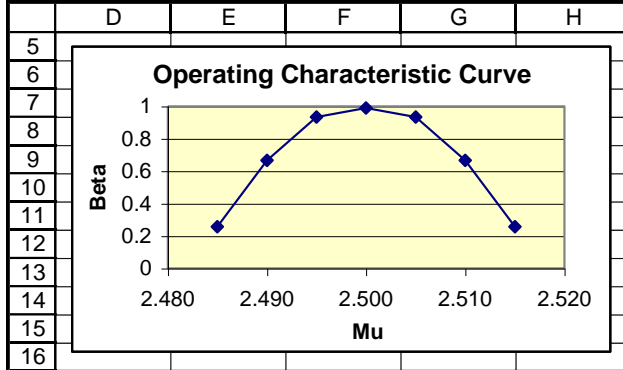
The operating characteristic curve.



10.88 p/a/m From exercise 10.84,

When $\mu = 2.485$, $\beta = 0.2578$ When $\mu = 2.505$, $\beta = 0.9345$
 When $\mu = 2.490$, $\beta = 0.6664$ When $\mu = 2.510$, $\beta = 0.6664$
 When $\mu = 2.495$, $\beta = 0.9345$ When $\mu = 2.515$, $\beta = 0.2578$
 When $\mu = 2.500$, $\beta = 0.9904$

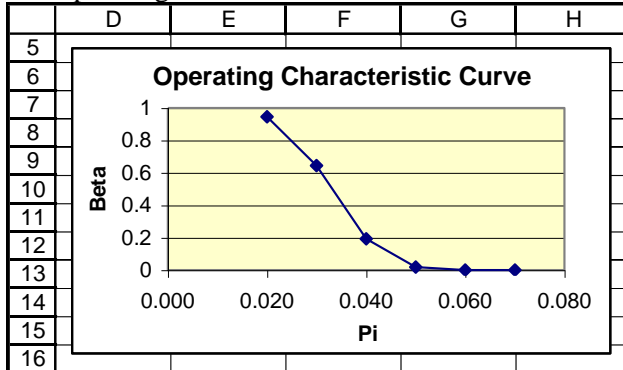
The operating characteristic curve is shown below:



10.89 p/a/m From exercise 10.86,

When $\pi = 0.02$, $\beta = 0.9463$ When $\pi = 0.05$, $\beta = 0.0179$
 When $\pi = 0.03$, $\beta = 0.6443$ When $\pi = 0.06$, $\beta = 0.0000$
 When $\pi = 0.04$, $\beta = 0.1922$ When $\pi = 0.07$, $\beta = 0.0000$

The operating characteristic curve is shown below:



CHAPTER EXERCISES

10.90 d/p/m

- a. For this situation, a left-tail test should be used since quality should now be improved. The appropriate null and alternative hypotheses are $H_0: \pi \geq 0.05$ and $H_1: \pi < 0.05$. A left-tail test is appropriate since, if the quality is improved, the proportion of defectives would be smaller than 0.05.
- b. For this situation, a two-tail test should be used. The appropriate null and alternative hypotheses are $H_0: \pi = 0.55$ and $H_1: \pi \neq 0.55$. A two-tail test is appropriate since it is a nondirectional statement that could be rejected by an extreme result in either direction.
- c. For this situation, a left-tail test should be used since a dealer would want to improve this value. The appropriate null and alternative hypotheses are $H_0: \pi \geq 0.70$ and $H_1: \pi < 0.70$. A left-tail test is appropriate since, if the dealer improved the pre-delivery mechanical checks, the proportion of cars having 3 or more mechanical problems in the first 4 months of ownership should decrease.

10.91 d/p/m H_0 : The employee has not taken drugs, and H_1 : The employee has taken drugs
 A Type I error will occur if we decide the employee has taken drugs but he really has not taken drugs.
 A Type II error will occur if we decide the employee has not taken drugs but he really has taken drugs.

10.92 p/a/m Null and alternative hypotheses:

$H_0: \mu \geq 1400$ (the bolts are genuine) and $H_1: \mu < 1400$ (the bolts are not genuine)

Level of significance: We will use the $\alpha = 0.05$ level in carrying out this left-tail test.

Test results: $\bar{x} = 1385$, $n = 20$ (known: $\sigma = 30$ and the population is normally distributed)

Calculated value of test statistic:
$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{1385 - 1400}{30/\sqrt{20}} = -2.24$$

Critical value: $z = -1.645$ (beneath the normal curve, the area to the left of $z = -1.645$ is 0.05).

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, it is possible that the bolts in the shipment might not be genuine since the mean tensile strength of the bolts is significantly less than 1400 pounds.

Given the information in this exercise, we could also use Excel worksheet template tmztest to obtain a solution. Just enter the values for n , \bar{x} , σ , and the hypothesized value for μ . The Excel printout is shown below, and the left-tail portion of the p-value section is in bold type for emphasis. The p-value for the test is 0.0127.

	A	B	C	D	E
1	1-Sample Z-Test, Known Sigma				
2					
3	<i>Sample Summary and Assumed Values:</i>		<i>Calculated Values:</i>		
4	Observed Sample Mean	1385.0000	Std. Error	6.7082	
5	Sample Size	20	z =	-2.2361	
6	Hypothesized Pop. Mean	1400.0000	<i>p-Value If the Test Is:</i>		
7	Assumed Pop. Std. Deviation	30.0000	Left-Tail	Two-Tail	Right-Tail
8			0.0127	0.0253	0.9873

10.93 p/a/m Null and alternative hypotheses:

$H_0: \mu \leq 45.4$ (mean productivity has not increased) and $H_1: \mu > 45.4$ (productivity has increased)

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 47.5$, $n = 30$ (known: $\sigma = 4.5$)

$$\text{Calculated value of test statistic: } z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{47.5 - 45.4}{4.5/\sqrt{30}} = 2.56$$

Critical value: $z = 2.33$ (beneath the normal curve, the area to the right of $z = 2.33$ is 0.01).

Decision rule: Reject H_0 if the calculated $z > 2.33$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.01 level, it appears the efficiency expert's efforts have been successful, since the mean productivity is now significantly more than 45.4 units per hour.

Given the information in this exercise, we could also use Excel worksheet template tmztest to obtain a solution. Just enter the values for n , \bar{x} , σ , and the hypothesized value for μ . The Excel printout is shown below, and the right-tail portion of the p-value section is in bold type for emphasis. The p-value for the test is 0.0053.

	A	B	C	D	E
1	1-Sample Z-Test, Known Sigma				
2					
3	<i>Sample Summary and Assumed Values:</i>		<i>Calculated Values:</i>		
4	Observed Sample Mean	47.5000	Std. Error	0.8216	
5	Sample Size	30	z =	2.5560	
6	Hypothesized Pop. Mean	45.4000	<i>p-Value If the Test Is:</i>		
7	Assumed Pop. Std. Deviation	4.5000	Left-Tail	Two-Tail	Right-Tail
8			0.9947	0.0106	0.0053

10.94 p/a/m

a. Null and alternative hypotheses:

$H_0: \mu = 3.13$ (The average family size in this city is the same as the U.S. average)

$H_1: \mu \neq 3.13$ (The average family size in this city is not the same as the U.S. average)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 3.40$, $s = 1.10$, $n = 40$

$$\text{Calculated value of test statistic: } t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{3.40 - 3.13}{1.10/\sqrt{40}} = 1.552$$

Critical values: $t = -2.023$ and $t = 2.023$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (40 - 1) = 39$.

Referring to the $0.05/2 = 0.025$ column and the 39th row of the t table, the critical values are $t = -2.023$ and $t = 2.023$.

Decision rule: Reject H_0 if the calculated $t < -2.023$ or > 2.023 , otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, there is no reason to believe that the average family size in this city is different from the national average of 3.13 persons. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance.

b. The 95% confidence interval for μ is: $\bar{x} \pm t \frac{s}{\sqrt{n}} = 3.40 \pm 2.023 \frac{1.10}{\sqrt{40}} = 3.40 \pm 0.352$,

or from 3.048 to 3.752. Since 3.13 is in the 95% confidence interval for μ found above, do not reject H_0 . This is the same conclusion that was reached in part a.

Given the information in this exercise, we can also use the Test Statistics and Estimators workbooks that accompany Data Analysis Plus. The t-test result and the 95% confidence interval are shown in the printouts below. The p-value for this two-tail test is 0.129. The lower and upper limits of the 95% confidence interval are 3.048 and 3.752, respectively.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	3.40	t Stat	1.552
4	Sample standard deviation	1.10	P(T<=t) one-tail	0.064
5	Sample size	40	t Critical one-tail	1.685
6	Hypothesized mean	3.13	P(T<=t) two-tail	0.129
7	Alpha	0.05	t Critical two-tail	2.023

	A	B	C	D	E
1	t-Estimate of a Mean				
2					
3	Sample mean	3.40	Confidence Interval Estimate		
4	Sample standard deviation	1.10	3.40	±	0.352
5	Sample size	40	Lower confidence limit		
6	Confidence level	0.95	Upper confidence limit		

10.95 p/a/m

a. Null and alternative hypotheses:

$H_0: \mu = 235,600$ (The average life insurance in this city is the same as the national average.)

$H_1: \mu \neq 235,600$ (The average life insurance in this city is not the same as the national average.)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 245,800$, $s = 25,500$, $n = 30$

Calculated value of test statistic: $t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{245,800 - 235,600}{25,500 / \sqrt{30}} = 2.191$

Critical values: $t = -2.045$ and $t = 2.045$ For this test, $\alpha = 0.05$ and d.f. $= (n - 1) = (30 - 1) = 29$.

Referring to the $0.05/2 = 0.025$ column and the 29th row of the t table, the critical values are $t = -2.045$ and $t = 2.045$.

Decision rule: Reject H_0 if the calculated $t < -2.045$ or > 2.045 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the average amount of life insurance in this city appears to be different from the national average of \$235,600.

b. The 95% confidence interval for μ is: $\bar{x} \pm t \frac{s}{\sqrt{n}} = 245,800 \pm 2.045 \frac{25,500}{\sqrt{30}} = 245,800 \pm 9521$,

or from \$236,279 to \$255,321. Since \$235,600 is not in the 95% confidence interval for μ found above, reject H_0 . This is the same conclusion that was reached in part a.

Given the information in this exercise, we can also use the Test Statistics and Estimators workbooks that accompany Data Analysis Plus. The t-test result and the 95% confidence interval are shown in the printouts below. The p-value for this two-tail test is 0.037.

The lower and upper limits of the 95% confidence interval are \$236,278 and \$255,322. Because they are not dependent on the printed t table and its inherent gaps between listed values, these confidence limits are more accurate than the ones calculated above.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	245800	t Stat	2.191
4	Sample standard deviation	25500	P(T<=t) one-tail	0.018
5	Sample size	30	t Critical one-tail	1.699
6	Hypothesized mean	235600	P(T<=t) two-tail	0.037
7	Alpha	0.05	t Critical two-tail	2.045

	A	B	C	D	E
1	t-Estimate of a Mean				
2					
3	Sample mean	245800	Confidence Interval Estimate		
4	Sample standard deviation	25500	245800		
5	Sample size	30	Lower confidence limit	±	9522
6	Confidence level	0.95	Upper confidence limit		236278

10.96 p/a/m Null and alternative hypotheses:

$H_0: \mu = 800$ (shipment meets company specifications)

$H_1: \mu \neq 800$ (shipment does not meet company specifications)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 805$, $n = 30$ (known: $\sigma = 12$)

Calculated value of test statistic: $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{805 - 800}{12/\sqrt{30}} = 2.28$

Critical values: $z = -1.96$ and $z = 1.96$

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the superintendent's complaint appears to be justified since the mean power consumption is significantly different from 800 watts.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this two-tail test, the p-value is 0.022.

	A	B	C	D
1	z-Test of a Mean			
2				
3	Sample mean	805	z Stat	2.28
4	Population standard deviation	12	P(Z<=z) one-tail	0.011
5	Sample size	30	z Critical one-tail	1.645
6	Hypothesized mean	800	P(Z<=z) two-tail	0.022
7	Alpha	0.05	z Critical two-tail	1.960

10.97 p/a/d Null and alternative hypotheses:

$H_0: \mu \geq 356$ (Mr. Jones is not too lenient with audits)

$H_1: \mu < 356$ (Mr. Jones is too lenient with audits)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 322$, $s = 90$, $n = 30$

Calculated value of test statistic: $t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{322 - 356}{90/\sqrt{30}} = -2.069$

Critical value: $t = -1.699$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (30 - 1) = 29$. Referring to the 0.05 column and the 29th row of the t table, the critical value is $t = -1.699$.

Decision rule: Reject H_0 if the calculated $t < -1.699$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the suspicions regarding Mr. Jones appear to be justified. The average amount of extra taxes collected by Mr. Jones appears to be less than \$356.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this left-tail test, the p-value is 0.024.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	322	t Stat	-2.07
4	Sample standard deviation	90.00	P(T<=t) one-tail	0.024
5	Sample size	30	t Critical one-tail	1.699
6	Hypothesized mean	356	P(T<=t) two-tail	0.048
7	Alpha	0.05	t Critical two-tail	2.045

10.98 p/a/m Null and alternative hypotheses:

$H_0: \mu \leq 0.40$ (the official's claim is correct) and $H_1: \mu > 0.40$ (the claim is not correct)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 0.46$, $s = 0.16$, $n = 35$

Calculated value of test statistic: $t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{0.46 - 0.40}{0.16/\sqrt{35}} = 2.219$

Critical value: $t = 1.691$ For this test, $\alpha = 0.05$ and d.f. = $(n - 1) = (35 - 1) = 34$. Referring to the 0.05 column and the 34th row of the t table, the critical value is $t = 1.691$.

Decision rule: Reject H_0 if the calculated $t > 1.691$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, we can reject the official's claim that the mean waiting time at exit booths from a toll road near the capital is no more than 0.40 minutes.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this right-tail test, the p-value is 0.017.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	0.46	t Stat	2.22
4	Sample standard deviation	0.16	P(T<=t) one-tail	0.017
5	Sample size	35	t Critical one-tail	1.691
6	Hypothesized mean	0.4	P(T<=t) two-tail	0.033
7	Alpha	0.05	t Critical two-tail	2.032

10.99 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.03$ (This region does not have more of a burglary problem than the nation.)

$H_1: \pi > 0.03$ (This region does have more of a burglary problem than the nation.)

Level of significance: $\alpha = 0.05$

Test results: $p = 18/300 = 0.06$, $n = 300$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.06 - 0.03}{\sqrt{0.03(1 - 0.03) / 300}} = 3.05$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, this region should be considered as having a burglary problem greater than the nation as a whole, since the percentage of households burglarized in this region is significantly larger than 3.0%. Using the standard normal table, $p\text{-value} = P(z > 3.05) = 1.0000 - 0.9989 = 0.0011$. From the $p\text{-value}$ perspective, we reject H_0 since $p\text{-value} = 0.0011$ is less than $\alpha = 0.05$.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this right-tail test, the $p\text{-value}$ is listed as 0.001.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.06	z Stat	3.05
4	Sample size	300	P(Z<=z) one-tail	0.001
5	Hypothesized proportion	0.03	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.002
7			z Critical two-tail	1.960

10.100 $p/a/m$ Null and alternative hypotheses:

$H_0: \pi \leq 0.08$ (no more Liberty families had refrigerators than the nation overall.)

$H_1: \pi > 0.08$ (more Liberty families had refrigerators than the nation overall.)

Level of significance: $\alpha = 0.01$

Test results: $p = 0.15$, $n = 120$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.15 - 0.08}{\sqrt{0.08(1 - 0.08) / 120}} = 2.83$

Critical value: $z = 2.33$

Decision rule: Reject H_0 if the calculated $z > 2.33$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.01 level, the percentage of Liberty families owning a "mechanical refrigerator" was significantly higher than the nation overall.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this right-tail test, the $p\text{-value}$ is listed as 0.002.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.15	z Stat	2.83
4	Sample size	120	P(Z<=z) one-tail	0.002
5	Hypothesized proportion	0.08	z Critical one-tail	2.326
6	Alpha	0.01	P(Z<=z) two-tail	0.005
7			z Critical two-tail	2.576

10.101 $p/a/m$ Null and alternative hypotheses:

$H_0: \pi = 0.30$ (the administrator's statement is correct) and $H_1: \pi \neq 0.30$ (statement is not correct)

Level of significance: $\alpha = 0.05$

Test results: $p = 0.35$, $n = 400$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.35 - 0.30}{\sqrt{0.30(1 - 0.30)/400}} = 2.18$

Critical values: $z = -1.96$ and $z = 1.96$

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96 , otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0 .

Decision: At the 0.05 level, the administrator's statement does not appear to be correct. Based on these results, the true proportion of emergency room patients that are not really in need of emergency treatment is not 0.30.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this two-tail test, the p-value is listed as 0.029.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.35	z Stat	2.18
4	Sample size	400	P(Z<=z) one-tail	0.015
5	Hypothesized proportion	0.30	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.029
7			z Critical two-tail	1.960

10.102 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.50$ (no more than half prefer the chunky version)

$H_1: \pi > 0.50$ (more than half prefer the chunky version)

Level of significance: $\alpha = 0.05$

Test results: $p = 58/100 = 0.58$, $n = 100$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.58 - 0.50}{\sqrt{0.50(1 - 0.50)/100}} = 1.60$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, we cannot conclude that this proportion is larger than the proportion that would tend to result from chance. Using the standard normal table, $p\text{-value} = P(z > 1.60) = 1.0000 - 0.9452 = 0.0548$. From the p-value perspective, we do not reject H_0 since $p\text{-value} = 0.0548$ is not less than $\alpha = 0.05$.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this right-tail test, the p-value is listed as 0.055.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.58	z Stat	1.60
4	Sample size	100	P(Z<=z) one-tail	0.055
5	Hypothesized proportion	0.50	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.110
7			z Critical two-tail	1.960

10.103 p/a/m Null and alternative hypotheses:

$H_0: \pi \leq 0.10$ (exterminator's claim is correct) and $H_1: \pi > 0.10$ (claim is not correct)

Level of significance: $\alpha = 0.05$

Test results: $p = 14/100 = 0.14$, $n = 100$

Calculated value of test statistic: $z = \frac{p - \pi_0}{\sigma_p} = \frac{0.14 - 0.10}{\sqrt{0.10(1 - 0.10)/100}} = 1.33$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.05 level, we have no reason to doubt the exterminator's claim. The proportion of homes the exterminator treats that have termite problems within one year after treatment is not significantly larger than 0.10.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this right-tail test, the p-value is listed as 0.091.

	A	B	C	D
1	z-Test of a Proportion			
2				
3	Sample proportion	0.14	z Stat	1.33
4	Sample size	100	P(Z<=z) one-tail	0.091
5	Hypothesized proportion	0.10	z Critical one-tail	1.645
6	Alpha	0.05	P(Z<=z) two-tail	0.182
7			z Critical two-tail	1.960

10.104 p/a/m Null and alternative hypotheses:

$H_0: \mu \geq 5$ (the chain's assertion is correct) and $H_1: \mu < 5$ (assertion is not correct)

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 4.6$, $s = 1.5$, $n = 40$

Calculated value of test statistic: $t = \frac{\bar{x} - \mu_0}{s_x} = \frac{4.6 - 5}{1.5/\sqrt{40}} = -1.687$

Critical value: $t = -2.426$ For this test, $\alpha = 0.01$ and d.f. = $(n - 1) = (40 - 1) = 39$. Referring to the 0.01 column and the 39th row of the t table, the critical value is $t = -2.426$.

Decision rule: Reject H_0 if the calculated $t < -2.426$, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0 .

Decision: At the 0.01 level, the evidence is not strong enough to dismiss the health club's contention that the mean number of pounds lost by members during the past month was at least 5 pounds. The sample mean weight loss could have been this low merely by chance.

Using the Test Statistics workbook that accompanies Data Analysis Plus, the corresponding printout is shown below. For this left-tail test, the p-value is listed as 0.050.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	4.6	t Stat	-1.69
4	Sample standard deviation	1.5	P(T<=t) one-tail	0.050
5	Sample size	40	t Critical one-tail	2.426
6	Hypothesized mean	5	P(T<=t) two-tail	0.100
7	Alpha	0.01	t Critical two-tail	2.708

10.105 p/a/d Null and alternative hypotheses: $H_0: \pi \geq 0.75$ and $H_1: \pi < 0.75$

The standard error of p can be calculated as: $\sigma_p = \sqrt{\frac{0.75(1 - 0.75)}{40}} = 0.0685$

We must first express the decision rule "Reject H_0 if $z < -1.645$ " in terms of p:

The sample proportion, p corresponding to $z = -1.645$ is $p = 0.75 - 1.645(0.0685) = 0.637$

The decision rule in terms of p will be: "Reject H_0 if $p < 0.637$."

a. Let $\pi = 0.75$:

$$\text{when } p = 0.637, z = \frac{p - \pi}{\sigma_p} = \frac{0.637 - 0.75}{0.0685} = -1.65$$

$$\text{and } \beta = P(z \geq -1.65) = 1.0000 - 0.0495 = 0.9505$$

b. Let $\pi = 0.70$:

$$\text{when } p = 0.637, z = \frac{p - \pi}{\sigma_p} = \frac{0.637 - 0.70}{0.0685} = -0.92$$

$$\text{and } \beta = P(z \geq -0.92) = 1.0000 - 0.1788 = 0.8212$$

c. Let $\pi = 0.65$:

$$\text{when } p = 0.637, z = \frac{p - \pi}{\sigma_p} = \frac{0.637 - 0.65}{0.0685} = -0.19$$

$$\text{and } \beta = P(z \geq -0.19) = 1.0000 - 0.4247 = 0.5753$$

d. Let $\pi = 0.60$:

$$\text{when } p = 0.637, z = \frac{p - \pi}{\sigma_p} = \frac{0.637 - 0.60}{0.0685} = 0.54$$

$$\text{and } \beta = P(z \geq 0.54) = 1.0000 - 0.7054 = 0.2946$$

e. Let $\pi = 0.55$:

$$\text{when } p = 0.637, z = \frac{p - \pi}{\sigma_p} = \frac{0.637 - 0.55}{0.0685} = 1.27$$

$$\text{and } \beta = P(z \geq 1.27) = 1.0000 - 0.8980 = 0.1020$$

- f. When $\pi = 0.75$, $1 - \beta = 1 - 0.9505 = 0.0495$ When $\pi = 0.70$, $1 - \beta = 1 - 0.8212 = 0.1788$
 When $\pi = 0.65$, $1 - \beta = 1 - 0.5753 = 0.4247$ When $\pi = 0.60$, $1 - \beta = 1 - 0.2946 = 0.7054$
 When $\pi = 0.55$, $1 - \beta = 1 - 0.1020 = 0.8980$

Using Minitab, note that the "Alternative Proportion" column refers to the assumed actual π . The Minitab results are much more accurate than the ones calculated above, largely due to our rounding in the quantities either leading to the calculation or resulting from it, including p, σ_p , and z.

Power and Sample Size

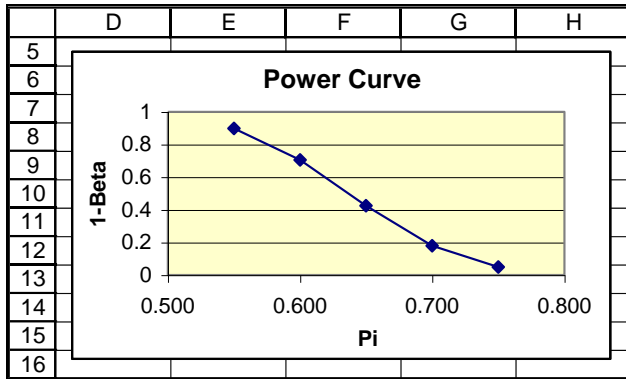
Test for One Proportion

Testing proportion = 0.75 (versus < 0.75)

Alpha = 0.05

Alternative Proportion	Sample Size	Power
0.750000	40	0.0500
0.700000	40	0.1937
0.650000	40	0.4336
0.600000	40	0.6853
0.550000	40	0.8667

Using Excel to chart the power curve. This plot graphs the power of the test = $1 - \beta$ = probability that the hypothesis test will correctly reject a false null hypothesis against the assumed value of π .



10.106 p/a/d

a. The null and alternative hypotheses are:

$H_0: \mu \geq 1600$ (the pet food company is not underfilling the packages)

$H_1: \mu < 1600$ (the pet food company is underfilling the packages)

This test can be carried out with a pocket calculator and formulas, but we will use the Test Statistics workbook that accompanies Data Analysis Plus. As shown in the printout below, the p-value for this left-tail test is 0.006. If the company were really putting an average of 1600 grams into the packages, there would be only a 0.006 probability of getting a sample mean this low.

Because p-value = 0.006 is less than $\alpha = 0.05$, the consumer agency will reject H_0 and conclude that the company is underfilling the packages. Perhaps the president of the company might prefer to use an α value that is numerically very small, such as $\alpha = 0.00001$, in order to force the conclusion that the null hypothesis would not be rejected.

	A	B	C	D
1	t-Test of a Mean			
2				
3	Sample mean	1591.7	t Stat	-2.65
4	Sample standard deviation	18.5	P(T<=t) one-tail	0.006
5	Sample size	35	t Critical one-tail	1.691
6	Hypothesized mean	1600	P(T<=t) two-tail	0.012
7	Alpha	0.05	t Critical two-tail	2.032

b. If we were relying on the pocket calculator and formulas, we would first have to express the decision rule for this test in terms of the sample mean: In a left-tail t-test at the 0.05 level, with $n = 35$, df will be $(35 - 1) = 34$ and the critical value of t will be $t = -1.690$. With $s = 18.5$ grams and $n = 35$, the standard error for the sample mean will be $18.5 / \sqrt{35} = 3.127$ grams.

The sample mean corresponding to the critical $t = -1.690$ will be $1600 - 1.690(3.127)$, or 1595.7154 grams, and the decision rule will be "Reject H_0 if $\bar{x} < 1595.7154$ grams."

We will bypass the pocket calculator and use Minitab to generate the power curve values for a range of assumed population means. These are: 1600, 1598, 1596, 1594, 1592, 1590, 1588, 1586, 1584, 1582, and 1580. The entries in the Difference column correspond to the difference between the assumed population mean and the value in the null hypothesis -- e.g., the -2.0000 entry corresponds to an assumed population mean of $1600 - 2.0000$, or 1598.

Power and Sample Size

1-Sample t Test

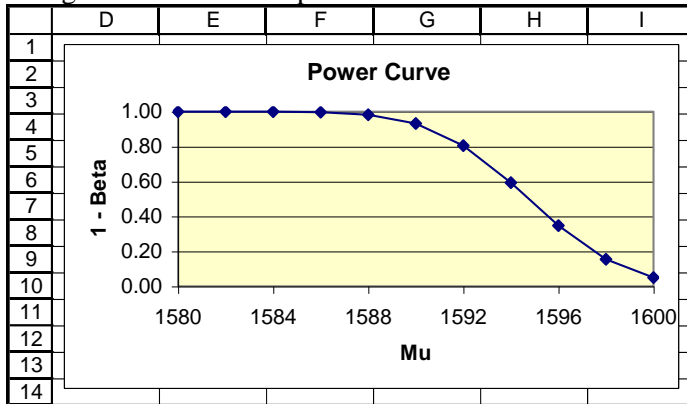
Testing mean = null (versus < null)

Calculating power for mean = null + difference

Alpha = 0.05 Sigma = 18.5

Difference	Sample Size	Power
0.0000	35	0.0500
-2.0000	35	0.1544
-4.0000	35	0.3478
-6.0000	35	0.5931
-8.0000	35	0.8057
-10.0000	35	0.9317
-12.0000	35	0.9828
-14.0000	35	0.9969
-16.0000	35	0.9996
-18.0000	35	1.0000
-20.0000	35	1.0000

Using Excel to chart the power curve.



10.107 p/a/d

- Shop-Mart should consider switching to Phipps bulbs. If the Phipps bulbs were really no better than the G&E, they would have had only a 0.012 probability of having this great an advantage in our tests just by chance.
- G&E might like to use the 0.005 level of significance in reaching a conclusion. Since the p-value is not less than 0.005, using the $\alpha = 0.005$ level would lead to the conclusion that the Phipps advantage in the test could have been merely due to chance.
- Phipps might like to use the 0.02 level of significance in reaching a conclusion. Since the p-value is less than 0.02, using the $\alpha = 0.02$ level would lead to the conclusion that the Phipps advantage in the test was not merely due to chance, and that the Phipps bulbs really are better.
- If the test had been two-tail instead of one-tail, the p-value would have been 0.024. We would have had to consider two tail areas instead of just one, and the mirror-image area on the other side would also have been 0.012. In this case, the p-value would have been $2(0.012) = 0.024$.

10.108 p/c/m The null and alternative hypotheses are $H_0: \mu \leq \$2.75$ and $H_1: \mu > \$2.75$.

Printout results are shown below for Data Analysis Plus and Minitab. In this right-tail test, the sample mean of \$3.30 exceeds the hypothesized mean of \$2.75 and the p-value is 0.001.

Since the p-value is less than the level of significance being used to reach a conclusion (0.025), we reject the null hypothesis and conclude that the new exhibit has increased the average contribution of exhibit patrons. If the new exhibit were no better than the old exhibit in attracting contributions, there would be only a 0.001 probability of obtaining a sample mean this large.

	A	B	C	D
1	t-Test: Mean			
2				
3				<i>Contrib</i>
4	Mean			3.3
5	Standard Deviation			0.861
6	Hypothesized Mean			2.75
7	df			29
8	t Stat			3.500
9	P(T<=t) one-tail			0.001
10	t Critical one-tail			2.045
11	P(T<=t) two-tail			0.002
12	t Critical two-tail			2.364

One-Sample T: Contrib

Test of mu = 2.75 vs mu > 2.75

Variable	N	Mean	StDev	SE Mean
Contrib	30	3.300	0.861	0.157

Variable	95.0% Lower Bound	T	P
Contrib	3.033	3.50	0.001

10.109 p/c/m The null and alternative hypotheses are $H_0: \pi = 0.25$ and $H_1: \pi \neq 0.25$.

Printout results are shown below for Data Analysis Plus. Of the 400 crimes in the sample, 30.25% involved a weapon. In this two-tail test, the p-value is 0.015, which is less than the 0.05 level of significance being used to reach a conclusion. We reject the null hypothesis and conclude that this city's experience is different from the nation as a whole in terms of the percent of violent crimes that involve a weapon. If the city were really the same as the rest of the nation, there would be only a 0.015 probability of obtaining a sample proportion this far away from 0.25.

	A	B	C	D
1	z-Test: Proportion			
2				
3				<i>Weapon</i>
4	Sample Proportion			0.3025
5	Observations			400
6	Hypothesized Proportion			0.25
7	z Stat			2.425
8	P(Z<=z) one-tail			0.008
9	z Critical one-tail			1.645
10	P(Z<=z) two-tail			0.015
11	z Critical two-tail			1.960

10.110 p/c/m The Data Analysis Plus printout below shows the 95% confidence interval for π as

0.2575 to 0.3475. Since the hypothesized value (0.25) is outside the interval, we conclude that this city's proportion must be some value other than 0.25. This is the same conclusion that was reached in exercise 10.109.

	A	B
1	z-Estimate: Proportion	
2		<i>Weapon</i>
3	Sample Proportion	0.3025
4	Observations	400
5	LCL	0.2575
6	UCL	0.3475

10.111 p/c/m The null and alternative hypotheses are $H_0: \mu \geq 3.5$ and $H_1: \mu < 3.5$.

Printout results are shown for Data Analysis Plus and Minitab. In this left-tail test, the sample mean of 3.293 ounces is less than the hypothesized mean (3.5000), but the p-value (0.059) is not less than the level of significance used to reach a conclusion (0.01), so we do not reject the null hypothesis. The new procedure has not significantly reduced the average amount of aluminum trimmed and recycled. At this level of significance, a sample mean this small could have happened by chance.

	A	B	C	D
1	t-Test: Mean			
2				
3				<i>Ounces</i>
4	Mean			3.293
5	Standard Deviation			0.764
6	Hypothesized Mean			3.5
7	df			34
8	t Stat			-1.605
9	P(T<=t) one-tail			0.059
10	t Critical one-tail			2.441
11	P(T<=t) two-tail			0.118
12	t Critical two-tail			2.728

One-Sample T: Ounces

Test of mu = 3.5 vs mu < 3.5

Variable	N	Mean	StDev	SE Mean
Ounces	35	3.293	0.764	0.129

Variable	95.0% Upper Bound	T	P
Ounces	3.511	-1.61	0.059

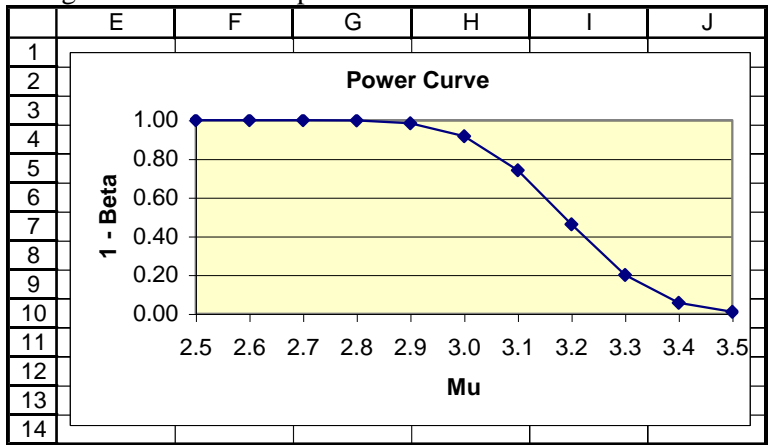
10.112 p/c/m Using Minitab to determine the power of the test for a selection of assumed values for the actual population mean, we obtain the results shown below. The selected assumed values range from 3.5 ounces (Difference = 0) to 2.5 ounces (Difference = -1.0).

Power and Sample Size
1-Sample t Test

Testing mean = null (versus < null)
Calculating power for mean = null + difference
Alpha = 0.01 Sigma = 0.764

Difference	Sample Size	Power
0.0	35	0.0100
-0.1	35	0.0568
-0.2	35	0.2008
-0.3	35	0.4619
-0.4	35	0.7411
-0.5	35	0.9176
-0.6	35	0.9834
-0.7	35	0.9980
-0.8	35	0.9998
-0.9	35	1.0000
-1.0	35	1.0000

Using Excel to chart the power curve.



10.113 p/c/m The null and alternative hypotheses are $H_0: \mu \leq 12,000$ hours and $H_1: \mu > 12,000$ hours. Printout results are shown below for Data Analysis Plus and Minitab. In this right-tail test, the sample mean of 12,070.38 hours exceeds the hypothesized mean (12,000) and the p-value is 0.282. Since the p-value is not less than the level of significance specified (0.025), we do not reject the null hypothesis. The new injection pumps may be no better than the ones already in use.

	A	B	C	D
1	t-Test: Mean			
2				
3				<i>Hours</i>
4	Mean			12070.38
5	Standard Deviation			856.20
6	Hypothesized Mean			12000
7	df			49
8	t Stat			0.581
9	P(T<=t) one-tail			0.282
10	t Critical one-tail			2.010
11	P(T<=t) two-tail			0.564
12	t Critical two-tail			2.312

One-Sample T: Hours

Test of mu = 12000 vs mu > 12000

Variable	N	Mean	StDev	SE Mean
Hours	50	12070	856	121

Variable	95.0% Lower Bound	T	P
Hours	11867	0.58	0.282

INTEGRATED CASES

THORNDIKE SPORTS EQUIPMENT

For 40 racquetball racquets, the null and alternative hypotheses are:

$H_0: \mu \geq 3.25$ (the Cromwell machine is not faster) and $H_1: \mu < 3.25$ (Cromwell machine is faster)

Using Minitab, we obtain the following results:

One-Sample T: RBRacq

Test of $\mu = 3.25$ vs $\mu < 3.25$

Variable	N	Mean	StDev	SE Mean
RBRacq	40	3.1507	0.2443	0.0386

Variable	95.0% Upper Bound	T	P
RBRacq	3.2158	-2.57	0.007

The p-value for this test is 0.007. If the population mean were exactly $\mu = 3.25$, the probability of obtaining a sample mean this small or smaller would be just 0.007. If the p-value of 0.007 is less than the level of significance being used to reach a conclusion, we will reject the null hypothesis. This p-value is very small, and it seems safe to conclude that the Cromwell machine is faster than the current models at stringing racquetball racquets.

For 40 tennis racquets, the null and alternative hypotheses are:

$H_0: \mu \geq 4.13$ (the Cromwell machine is not faster) and $H_1: \mu < 4.13$ (Cromwell machine is faster)

Using Minitab, we obtain the following results:

One-Sample T: TennRacq

Test of $\mu = 4.13$ vs $\mu < 4.13$

Variable	N	Mean	StDev	SE Mean
TennRacq	40	4.0110	0.3377	0.0534

Variable	95.0% Upper Bound	T	P
TennRacq	4.1010	-2.23	0.016

The p-value for this test is 0.016. If the population mean were exactly $\mu = 4.13$, the probability of obtaining a sample mean this small or smaller would be just 0.016. If the p-value of 0.016 is less than the level of significance being used to reach a conclusion, we will reject the null hypothesis. This p-value is very small, and it seems safe to conclude that the Cromwell machine is faster than the current models at stringing tennis racquets.

From the results obtained above, Ted can be very confident that the Cromwell machine is faster than the current models at stringing both racquetball and tennis racquets. The tests appear to warrant purchase of the Cromwell machine.

SPRINGDALE SHOPPING SURVEY

1a. through 1d.

All of the desired information for Springdale Mall is provided within the t-test printout below:

```
One-Sample T: SPRILIKE
Test of mu = 3 vs mu not = 3
Variable      N      Mean      StDev      SE Mean
SPRILIKE      150     4.0867     0.7766     0.0634

Variable      90.0% CI                T      P
SPRILIKE      ( 3.9817, 4.1916)      17.14  0.000
```

- Yes, this area seems well regarded by the respondents. Recall that numerically higher scores are better.
- In testing $H_0: \mu_7 = 3.0$ versus $H_1: \mu_7 \neq 3.0$, the p-value (0.000, rounded to three decimal places) is less than 0.10, the level of significance specified for the test. We reject the null hypothesis that the population mean is equal to 3.0.
- The 90% confidence interval for μ_7 is shown in the printout. The hypothesized value (3.0) falls outside the 90% confidence interval. This is consistent with the hypothesis test result using the 0.10 level of significance.
- As shown above, the p-value for the hypothesis test is 0.000 (to three decimal places).

1e. All of the desired information for Downtown is provided within the t-test printout below:

```
One-Sample T: DOWNLIKE
Test of mu = 3 vs mu not = 3
Variable      N      Mean      StDev      SE Mean
DOWNLIKE      150     3.5200     0.9392     0.0767

Variable      90.0% CI                T      P
DOWNLIKE      ( 3.3931, 3.6469)      6.78  0.000
```

- Yes, this area seems well regarded by the respondents, but less so than Springdale Mall. Recall that numerically higher scores are better.
- In testing $H_0: \mu_8 = 3.0$ versus $H_1: \mu_8 \neq 3.0$, the p-value (0.000, rounded to three decimal places) is less than 0.10, the level of significance specified for the test. We reject the null hypothesis that the population mean is equal to 3.0.
- The 90% confidence interval for μ_8 is shown in the printout. The hypothesized value (3.0) falls outside the 90% confidence interval. This is consistent with the hypothesis test result using the 0.10 level of significance.
- As shown above, the p-value for the hypothesis test is 0.000 (to three decimal places).

1f. All of the desired information for West Mall is provided within the t-test printout below:

```
One-Sample T: WESTLIKE
Test of mu = 3 vs mu not = 3
Variable      N      Mean      StDev      SE Mean
WESTLIKE      150     3.2467     1.0423     0.0851

Variable      90.0% CI                T      P
WESTLIKE      ( 3.1058, 3.3875)      2.90  0.004
```

- Yes, this area seems well regarded by the respondents, but less so than Springdale Mall and Downtown. Recall that numerically higher scores are better.
- In testing $H_0: \mu_9 = 3.0$ versus $H_1: \mu_9 \neq 3.0$, the p-value (0.004) is less than 0.10, the level of significance specified for the test. We reject the null hypothesis that the population mean is equal to 3.0.

- The 90% confidence interval for μ_9 is shown in the printout. The hypothesized value (3.0) falls outside the 90% confidence interval. This is consistent with the hypothesis test result using the 0.10 level of significance.
- As shown above, the p-value for the hypothesis test is 0.004.

2. Shown below are the Minitab counts for variables 10 through 17:

BSTEXCHG	Count	BSTQUALI	Count	BSTPRICE	Count	BSTVARIE	Count
1	72	1	93	1	22	1	123
2	23	2	43	2	16	2	18
3	21	3	6	3	99	3	5
4	34	4	8	4	13	4	4
N=	150	N=	150	N=	150	N=	150

BSTHELP	Count	BSTHOURS	Count	BSTCLEAN	Count	BSTBARGN	Count
1	64	1	109	1	120	1	48
2	42	2	9	2	10	2	33
3	10	3	19	3	10	3	55
4	34	4	13	4	10	4	14
N=	150	N=	150	N=	150	N=	150

2a. through 2c. Tests for Springdale Mall, variables 10 through 17:

BSTEXCHG

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	72	116	0.620690	(0.532391, 0.708988)	6.57	0.000

BSTQUALI

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	93	142	0.654930	(0.576739, 0.733120)	8.14	0.000

BSTPRICE

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	22	137	0.160584	(0.099105, 0.222063)	-4.28	0.000

BSTVARIE

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	123	146	0.842466	(0.783373, 0.901559)	13.06	0.000

BSTHELP

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	64	116	0.551724	(0.461223, 0.642225)	5.00	0.000

BSTHOURS

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	109	137	0.795620	(0.728096, 0.863145)	11.49	0.000

BSTCLEAN

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	120	140	0.857143	(0.799178, 0.915107)	13.16	0.000

BSTBARGN

Test of p = 0.333 vs p not = 0.333

Sample	X	N	Sample p	95.0% CI	Z-Value	P-Value
1	48	136	0.352941	(0.272625, 0.433257)	0.49	0.622

These tests can be summarized as shown below. Note that n refers to the number of persons who selected one of the three shopping areas and p = the proportion of those persons who expressed a choice who selected Springdale Mall as the “best” location associated with that variable.

The rightmost column shows the p-value for the test of $H_0: \pi = 0.333$, whether the population proportion selecting Springdale Mall could be 0.333.

variable number	variable name	n	p	z	p-value
10	BSTEXCHG	116	0.621	6.57	0.000
11	BSTQUALI	142	0.655	8.14	0.000
12	BSTPRICE	137	0.161	-4.28	0.000
13	BSTVARIE	146	0.842	13.06	0.000
14	BSTHELP	116	0.552	5.00	0.000
15	BSTHOURS	137	0.796	11.49	0.000
16	BSTCLEAN	140	0.857	13.16	0.000
17	BSTBARGN	136	0.353	0.49	0.622

Of variables 10 through 17, and using the $\alpha = 0.05$ level of significance, we would reject $H_0: \pi = 0.333$ for all except one: variable 17 (BSTBARGN).

- 2d. Springdale Mall is the strongest of the three areas in all but two of these eight attributes. The only attributes for which it is not the "best-fit" are best prices and bargain sales.

BUSINESS CASE

PRONTO PIZZA (A)

We must first create a new variable called Tot_Time, which represents the total amount of time from the call being received to the delivery being made. It will be the total of Prep_Time, Wait_Time, and Travel_Time, and it is the time to which the guarantee would be applied.

1. In examining whether the population average for Tot_Time might be greater than 25 minutes, our null and alternative hypotheses are $H_0: \mu \leq 25$ and $H_1: \mu > 25$. We will use $\alpha = 0.05$ as the level of significance for this right-tail test. The Minitab printout is shown below. Because p-value = 0.104 is not less than the 0.05 level of significance for the test, we fail to reject H_0 and we conclude that the mean delivery time could be no more than 25 minutes.

One-Sample T: Tot_Time
Test of mu = 25 vs > 25

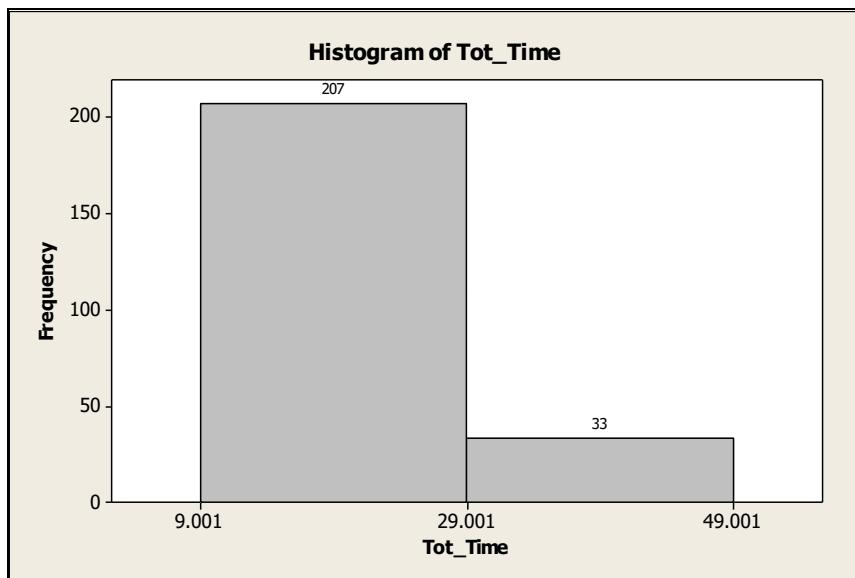
Variable	N	Mean	StDev	SE Mean	95% Lower Bound	T	P
Tot_Time	240	25.3205	3.9249	0.2534	24.9021	1.26	0.104

2. One approach is to sort the existing data in order of size and manually determine the percentage of cases in which Tot_Time was 29 minutes or less. This arrangement is shown below. An alternative is to use a Minitab histogram with the cutpoints set at 9.001, 29.001, and 49.001. The first bar in the histogram will include values that are at least 9.001, but less than 29.001, so this bar will include all the cases for which Tot_Time was 29.00 minutes or less. As we see in the histogram, Tot_Time was 29.00 minutes or less in 207 out of the 240 deliveries. This is a “success” percentage of 86.25%, but this means that Pronto failed to meet the 29.00-minute deadline in 13.75% of its deliveries. On this basis, it does not appear that Pronto will meet its requirement of failing to meet the guarantee no more than 5% of the time.

Data Display

Tot_Time

16.90	17.87	18.53	19.20	19.39	19.49	19.90	19.97	20.03
20.13	20.32	20.41	20.41	20.53	20.69	20.78	20.79	20.80
20.87	20.90	20.98	21.03	21.05	21.07	21.21	21.27	21.38
21.53	21.54	21.68	21.70	21.73	21.73	21.79	21.81	21.83
21.89	21.91	21.99	22.03	22.07	22.11	22.18	22.20	22.24
22.25	22.29	22.32	22.33	22.35	22.41	22.45	22.47	22.71
22.77	22.79	22.79	22.84	22.85	22.85	22.86	22.87	22.89
22.91	22.91	23.00	23.04	23.04	23.08	23.16	23.18	23.22
23.26	23.27	23.28	23.31	23.32	23.39	23.43	23.44	23.45
23.47	23.52	23.53	23.58	23.61	23.62	23.64	23.72	23.78
23.79	23.80	23.80	23.90	23.96	23.97	24.00	24.01	24.03
24.07	24.15	24.19	24.20	24.25	24.30	24.33	24.39	24.40
24.41	24.41	24.42	24.43	24.45	24.48	24.50	24.50	24.52
24.66	24.67	24.69	24.71	24.73	24.76	24.76	24.82	24.83
24.83	24.84	24.88	24.88	24.89	24.91	24.97	25.01	25.12
25.14	25.26	25.33	25.35	25.38	25.44	25.45	25.46	25.48
25.56	25.61	25.62	25.65	25.67	25.69	25.71	25.72	25.75
25.76	25.79	25.81	25.85	25.86	25.87	25.92	25.98	25.98
26.00	26.06	26.16	26.25	26.32	26.40	26.42	26.48	26.49
26.51	26.52	26.53	26.61	26.64	26.74	26.75	26.77	26.78
26.79	26.87	26.89	26.95	26.96	27.24	27.27	27.40	27.43
27.46	27.47	27.48	27.49	27.71	27.87	27.91	27.97	28.12
28.25	28.44	28.56	28.59	28.66	28.78	28.79	28.86	28.87
29.06	29.36	29.43	29.50	29.65	29.67	29.74	29.97	29.97
30.20	30.31	30.63	30.69	30.79	30.89	31.76	31.90	31.95
32.42	32.55	32.88	33.02	33.05	33.58	34.04	34.12	34.31
34.72	36.48	37.30	39.42	40.22	46.00			



- Comparing the average delivery times for different days of the week, Monday (code 3) has the shortest mean delivery time (23.886 minutes) and Saturday (code 6) has the longest mean delivery time (27.816 minutes). It appears that the day of the week does have an effect on the average time a customer will have to wait for his or her pizza. Of particular note is the relatively high standard deviation for the Saturday delivery times and the fact that the third quartile for this day is 29.665 minutes. On Saturdays, 25% of the deliveries require at least 29.665 minutes, a time that itself exceeds the desired guaranteed delivery time of 29 minutes. Friday delivery times also have a relatively high mean and standard deviation. In addition, the Friday third quartile value of 28.473 minutes is very high, nearly as great as the 29 minutes for the planned guarantee.

Descriptive Statistics: Tot_Time

Variable	Day	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Tot_Time	1	32	23.886	0.600	3.397	17.870	22.068	23.105	26.135
	2	32	25.054	0.553	3.126	20.530	23.143	24.500	25.793
	3	32	24.453	0.452	2.557	16.900	22.857	24.425	25.708
	4	32	23.928	0.399	2.255	20.030	22.335	23.895	25.573
	5	40	26.541	0.595	3.764	19.390	23.705	26.360	28.473
	6	40	27.816	0.894	5.653	20.410	24.185	26.245	29.665
	7	32	24.637	0.622	3.519	19.490	21.870	24.415	26.393

Variable	Day	Maximum
Tot_Time	1	32.880
	2	34.040
	3	29.970
	4	28.790
	5	34.310
	6	46.000
	7	33.050

4. As shown in the printout below, the longest delivery times are associated with the 5:00-5:59 hour (mean time = 26.545 minutes) and the shortest tend to be associated with the 11:00-11:59 hour (mean time = 24.703 minutes). The third quartile for the 5:00-5:59 hour is 29.378 minutes, which exceeds the 29-minute planned guarantee, as does the third quartile for the 7:00-7:59 hour (29.688 minutes). For orders placed during each of these hours, at least 25% of the deliveries will take longer than the planned guarantee.

Descriptive Statistics: Tot_Time

Variable	Hour	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Tot_Time	4	30	24.926	0.712	3.898	18.530	22.790	23.990	25.805
	5	30	26.545	0.714	3.910	20.410	22.900	26.070	29.378
	6	30	25.760	0.643	3.523	20.030	23.180	25.500	27.523
	7	30	25.851	0.761	4.168	19.200	23.155	25.660	29.688
	8	30	24.777	0.831	4.549	20.790	22.313	23.700	25.830
	9	30	25.169	0.823	4.507	16.900	22.645	24.465	26.550
	10	30	24.833	0.515	2.818	19.900	22.883	24.695	25.765
	11	30	24.703	0.697	3.818	17.870	22.440	23.920	27.310

Variable	Hour	Maximum
Tot_Time	4	39.420
	5	34.720
	6	34.310
	7	34.040
	8	46.000
	9	40.220
	10	31.900
	11	36.480

5. Based on the preceding analyses, Pronto Pizza may wish to increase its guaranteed time to a value slightly higher than 29 minutes. Another possibility, albeit one that could cause confusion among customers, is to guarantee 29-minute delivery only during days and/or times other than those cited previously. Because Tony will not be able to meet his 29-minute guarantee 95% of the time, as desired, he may wish to either increase the time specified in the guarantee or offer something less expensive than a free pizza if the guarantee is not met -- perhaps a free side order of bread sticks or a discount coupon for customers who have to wait longer than 29 minutes.

